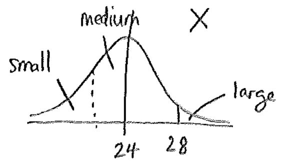
CIMT Statistics p170 Ex 8.7.

1. X = mass of plum

$X \sim N(24, 5^2)$



a) $P(\text{large}) = P(X > 28)$

$= P(Z > \frac{28-24}{5})$

$= P(Z > 0.8)$

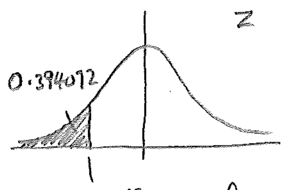
$= 0.211855$ from normCdf(0.8, 9E99)

$\approx \underline{0.2119}$ (4dp)

b) so small + medium amount to $1 - 0.211855$

$= 0.788145$

so 50% of $0.788145 = 0.394072$.



from invNorm(0.394072)

so $\frac{x - 24}{5} = -0.268721$

$x = 24 - 5 \times 0.268721$

$x = 22.6564$

$\therefore$ upper limit of "small plums" is $\underline{22.7g}$ (1dp)

2. X = demand for videos

$X \sim N(50, 10^2)$

a) $P(X > 65)$

$= P\left(Z > \frac{65-50}{10}\right)$

$= P(Z > 1.5)$

$= 0.066807$ from normCdf$(1.5, 9E99)$

$\approx \underline{0.0668}$ (4dp)

b) $P(X < 40)$

$= P\left(Z < \frac{40-50}{10}\right)$

$= P(Z < -1)$

$= 0.158655$

let Y = no. days less than 40

$Y \sim B(7, 0.158655)$

so $P(Y > 3) = P(4 \leq Y \leq 7)$

$= 0.014798$ from binomCdf$(7, 0.158655, 4, 7)$

$\approx \underline{0.0148}$ (4dp)

c) if demand increases by 25%.

then $X \sim N(50 \times 1.25, 10^2)$

$X \sim N(62.5, 10^2)$

now $P(X > 65) = P\left(Z > \frac{65-62.5}{10}\right)$

$= P(Z > 0.25)$

≈ 0.401294 from normCdf$(0.25, 9E99)$

$\approx \underline{0.4013}$ (4dp)

3. X = no. 6's scored.

assume die is fair

$X \sim B(200, \frac{1}{6})$

$P(X>40) = P(X \geq 41)$

$= 0.089425$ from binomCdf $(200, \frac{1}{6}, 41, 200)$ [exactly]

$\approx \underline{0.0894}$ (4dp)

OR by approximating with Normal:

$E(X) = \frac{200}{6}$

$Var(X) = \frac{200}{6} \times \frac{5}{6} = \frac{1000}{36}$

now $np > 5$ and $nq > 5$ so Normal approx is valid.

let Y = approx for X

$Y \sim N(\frac{200}{6}, \frac{1000}{36})$

$P(X>40) \doteqdot P(Y>40.5)$ by continuity correction

$= P\left(Z > \frac{40.5 - \frac{200}{6}}{\sqrt{\frac{1000}{36}}}\right)$

$= P(Z > 1.35978)$

$= 0.08695...$

$= \underline{0.0870}$ (4dp)

5% tail of $B(200, \frac{1}{6})$ distribution

<u>exactly</u>

on TI-Nspire:

nSolve (binomCdf $(200, \frac{1}{6}, n, 200) = 0.05, n$)

gives 42

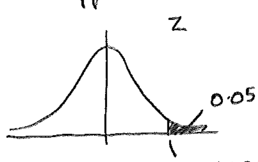
ie, $P(X \geq 42) = 0.05$

Hence he should discard if more than 41 sixes out of 200 rolls.

or, on TI-Nspire:

invBinom $(0.95, 200, \frac{1}{6})$ gives 42.

OR Normal approximation



1.64485 by invNorm (0.95)

so $\frac{y - \frac{200}{6}}{\sqrt{\frac{1000}{36}}} = 1.64485$

$y = \frac{200}{6} + 1.644485 \times \sqrt{\frac{1000}{36}}$

$y = 42.0025$

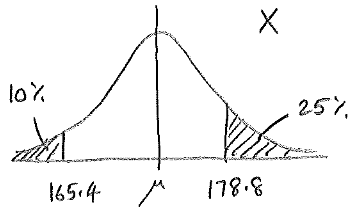
so $P(Y > 42.0025) = 0.05$

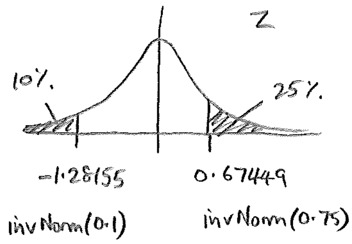
$\Rightarrow$ after continuity correction $P(X>41) = 0.05$

And hence obtain 42 or more 6's would reject dice.

4. X = heights of males

$X \sim N(\mu, \sigma)$





so $\dfrac{165.4 - \mu}{\sigma} = -1.28155$

and $\dfrac{178.8 - \mu}{\sigma} = 0.67449$

using linSolve $\left(\begin{cases} 165.4 - m = -1.28155s \\ 178.8 - m = 0.67449s \end{cases}, \{m, s\} \right)$ (where $m \Leftrightarrow \mu$, $s \Leftrightarrow \sigma$)

gives $\mu = 174.179$

$\sigma = 6.85057$

$\therefore$ mean height ≈ 174.2 cm, mean st. dev $= 6.85$ cm
(1dp) (2dp)

so $P(X > 183) = P\left(Z > \dfrac{183 - 174.2}{6.85}\right)$

$= P(Z > 1.28758)$

$\doteqdot 0.098947$

$= 0.0989$ (4dp)

5. X = arrive time, in mins after 8am

$X \sim N(0, 6^2)$

a) $P(\text{miss train}) = P(X > 5)$

$= P\left(Z > \frac{5-0}{6}\right)$

$= P\left(Z > \frac{5}{6}\right)$

$\doteqdot 0.202328\ldots$ from normCdf$\left(\frac{5}{6}, 9E99\right)$

$= \underline{0.2023}$ (4dp)

b) let Y = no. days miss train

$Y \sim B(5, 0.2023)$

$P(Y=1) = 0.409566$ from binomPdf$(5, 0.2023, 1)$

$\approx \underline{0.4096}$ (4dp)

c) 46 weeks = 230 days.

let W = no. days miss train

$W \sim B(230, 0.2023)$

$P(W < 35) = P(W \leq 34)$

$= 0.0211545\ldots$ using binomCdf$(230, 0.2023, 0, 34)$

$\approx \underline{0.0212}$ (4dp)

OR using normal approximation for W

let V = normal approx. for W

$V \sim N(230 \times 0.2023, 230 \times 0.2023 \times (1 - 0.2023))$

$V \sim N(46.5355, 37.1201)$ as $np > 5$, and $nq > 5$, approximation is valid ✓

so $P(W < 35) \doteqdot P(V < 34.5)$ by c.c.

$= P\left(Z < \frac{34.5 - 46.5355}{\sqrt{37.1201}}\right)$

$= P(Z < -1.97542)$

$= 0.02411\ldots$

$= \underline{0.0241}$ (4dp)

9. a) let X = no. adjustments per hour

$X \sim Po(8)$

Poisson used here with confidence as a constant rate per hour. We assume adjustments are independent of one another

i) $P(X \leq 5) = 0.191236...$ from poissCdf(8,0,5)

≈ 0.1912 (4dp)

ii) let Y = no. adjustments in 8 hour shift

$Y \sim Poi(64)$

$P(Y \geq 70) = 1 - P(Y \leq 69)$

$= 1 - 0.75759$ by poissCdf(64,0,69)

$= 0.24241...$

≈ 0.2424 (4dp)

OR by normal approximation.

let W be approx to Y

$W \sim N(64, 64)$ approximation valid as $\lambda > 10$

so $P(Y \geq 70) \doteq P(W > 69.5)$ by c.c.

$= P\left(Z > \frac{69.5 - 64}{8}\right)$

$= P(Z > 0.6875)$

$= 0.245884...$

$= 0.2459$ (4dp)

b) X = no. failing to germinate

$X \sim B(20, 0.15)$

i) $P(X \geq 5) = 0.170153...$ from binomCdf(20,0.15,5,20)

≈ 0.1702 (4dp)

ii) $P(\text{at least 17 germinated}) = P(X \leq 3)$

$= 0.647725...$ from binomCdf(20,0.15,0,3)

≈ 0.6477 (4dp)

Binomial would be used with confidence here if we assume that all of the tennants had equally good growing ground and they cared for the plants in an equal way. This, however, is unlikely, so these probabilities are approximate at best if one were to be selecting a single tennant at random. However, for one chosen tennant, depending on how they cared for the plants, the value of 0.15 may increase or decrease.