

p92 Ex4B no. 1

$X = \text{score on D6}$

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\text{or } P(X=x) = \begin{cases} \frac{1}{6} & x=1,2,3,4,5,6 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Var}(X) = E(X^2) - E(X)^2 \quad \text{now } E(X) = 3.5, \text{ by symmetry.}$$

$$\begin{aligned} E(X^2) &= \sum x^2 P(X=x) \\ &= \frac{1}{6} (1^2 + 2^2 + \dots + 6^2) \\ &= \frac{1}{6} \times 91 \end{aligned}$$

$$\begin{aligned} \text{so } \text{Var}(X) &= \frac{91}{6} - \left(\frac{7}{2}\right)^2 \\ &= \frac{35}{12} \end{aligned}$$

$$\text{so stdev of } X = \sqrt{\frac{35}{12}}$$

$$\approx \underline{\underline{1.7078}} \quad (4dp)$$

Ex 4B no. 2

a)

x	2	3	4	5	...	12
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$...	$\frac{1}{36}$

$$\text{or } P(X=x) = \begin{cases} \frac{x-1}{36} & x=2,3,4,5,6,7 \\ \frac{13-x}{36} & x=8,9,10,11,12 \end{cases}$$

$$E(X) = 7$$

$$E(X^2) = \sum x^2 P(X=x)$$

$$= \frac{329}{6} \quad \text{from Nspire}$$

$$\text{Var}(X) = E(X^2) - E^2(X)$$

$$= \frac{329}{6} - 7^2$$

$$= \frac{35}{6}$$

$$\text{so } \sigma_X = \sqrt{\frac{35}{6}}$$

$$\approx \underline{\underline{2.4152}} \quad (4dp)$$

b)

x	0	1	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$\text{or } P(X=x) = \begin{cases} \frac{1}{8} & x=0,3 \\ \frac{3}{8} & x=1,2 \end{cases}$$

$$E(X) = \frac{3}{2}$$

$$E(X^2) = \sum x^2 P(X=x)$$

$$= 3$$

$$\text{Var}(X) = E(X^2) - E^2(X)$$

$$= 3 - \left(\frac{3}{2}\right)^2$$

$$= \frac{3}{4}$$

$$\text{so } \sigma_X = \sqrt{\frac{3}{4}}$$

$$\approx \underline{\underline{0.8660}} \quad (4dp)$$

Ex4B no. 3 $X = \text{no. girls in the team of 3.}$

x	0	1	2	3
$P(X=x)$	$\frac{6}{210}$	$\frac{72}{210}$	$\frac{108}{210}$	$\frac{24}{210}$

$$P(X=0) = P(\text{all boys})$$

$$= \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5}$$

$$= \frac{6}{210}$$

3 boys
4 girls
7

$$P(X=1) = P(GBB) + P(BGB) + P(BBG)$$

$$= \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} + \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5} + \frac{3}{7} \times \frac{2}{6} \times \frac{4}{5}$$

$$= \frac{72}{210}$$

$$P(X=2) = P(GGB) + P(GBG) + P(BGG)$$

$$= 3 \times \frac{4 \times 3 \times 3}{210}$$

$$= \frac{108}{210}$$

$$P(X=3) = P(\text{all girls})$$

$$= \frac{4 \times 3 \times 2}{210}$$

$$= \frac{24}{210}$$

check - does $\sum P(X=x) = 1$?

$$108 + 72 + 24 + 6 = 210 \checkmark$$

$$E(X) = \sum x P(X=x)$$

$$= \frac{12}{7} \quad \text{from Nspire}$$

$$E(X^2) = \frac{24}{7}$$

$$\text{Var}(X) = E(X^2) - E^2(X)$$

$$= \frac{24}{7} - \left(\frac{12}{7}\right)^2$$

$$= \frac{24}{49}$$

$$\text{so } \sigma_X = \sqrt{\frac{24}{49}}$$

$$\approx 0.6999 \text{ (4dp)}$$