p111 Ex5B. no. 1

1. X = no. wickets taken in 10 overs

$X \sim B(10, \frac{1}{8})$

$P(X=0) = 0.263076...$

$= \underline{\underline{0.2631}}$ (4dp) from binompdf$(10, \frac{1}{8}, 0)$

Ex 5B no. 2

X = no. tails on n tosses of a coin

$X \sim B(n, \frac{1}{2})$

we want $P(X \geq 1) > 0.99$

$$1 - P(X=0) > 0.99$$
$$1 > 0.99 + P(X=0)$$
$$1 - 0.99 > P(X=0)$$
$$0.01 > P(X=0)$$
$$\frac{1}{100} > {}^nC_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^n$$
$$\frac{1}{100} > 1 \times 1 \times \frac{1}{2^n}$$
$$\frac{1}{100} > \frac{1}{2^n} \quad \times 100$$
$$1 > \frac{1}{2^n} \times 100 \quad \times 2^n$$
$$2^n > 100$$

now $2^6 = 64$ and $2^7 = 128$, so $n \geq 7$

so the smallest integer value of n so that $2^n > 100$ is $n = 7$.

so we need at least 7 tosses of the coin.

Ex 5 B no. 3

$X \sim B(11, p)$

$P(X=8) = {}^{11}C_8 \, p^8 (1-p)^3$

$P(X=7) = {}^{11}C_7 \, p^7 (1-p)^4$

so $\quad {}^{11}C_8 \, p^8 (1-p)^3 = {}^{11}C_7 \, p^7 (1-p)^4$

$\div p^7 (1-p)^3$

$${}^{11}C_8 \, p = {}^{11}C_7 \, (1-p)$$

$$165 p = 330(1-p)$$

$$495 p = 330$$

$$p = \frac{330}{495}$$

$$\underline{\underline{p = \frac{2}{3}}}.$$

Ex5B no.4.

no. boys.	0	1	2	3
freq.	13	34	40	13.

a) total boys $= 0\times13 + 1\times34 + 2\times40 + 3\times13$
$= 153.$

total children $= 100\times3$
$= 300$

$\therefore P(\text{boy is born}) = \frac{153}{300}$
$= \frac{51}{100}$
$= 0.51$

b) X = no. of boys in a family of 3 children

$X \sim B(3, 0.51)$

$P(X=2) = 0.382347\ldots$ by binompdf$(3, 0.51, 2)$

Y = no. families with 2 boys

$Y \sim B(100, 0.382347\ldots)$

so $E(Y) = 100 \times 0.382347$
≈ 38.2347

So we would expect about 38 or 39 families to have 2 boys and 1 girl.

Ex 5B no. 5.

$X =$ no. questions answered correctly by guessing

a) $X \sim B(20, \frac{1}{5})$

must include this assumption.

b) $E(X) = 20 \times \frac{1}{5} = 4$

$Var(X) = 20 \times \frac{1}{5} \times \frac{4}{5} = \frac{16}{5}$

$= 3\frac{1}{5}$.

c) $P(X \geq 10) = 0.0025948\ldots$ from binomcdf$(20, \frac{1}{5}, 10, 20)$

$= 0.0026$ (4dp)

Ex 5B no. 7.

$P(\text{pass test}) = 0.8$

X = number of students who pass test

$X \sim B(18, 0.8)$

we seek maximum value of $P(X=x)$. This maximum will be near the mean (or expected value)

now $E(X) = 18 \times 0.8$
$= 14.4$

so $P(X=14) = 0.2153$
$P(X=15) = 0.2297$

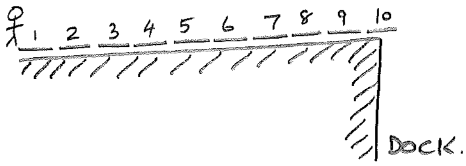
so most likely value is $\underline{\underline{15}}$.

Ex 5B no. 8

a) P(fall in dock on 10^{th} step)

= P(walked forwards 10 steps after taking 10 paces)

$= \underbrace{0{\cdot}5 \times 0{\cdot}5 \times 0{\cdot}5 \times \ldots \times 0{\cdot}5}_{\text{10 steps forwards}}$



$= (0{\cdot}5)^{10}$

$= 0{\cdot}0009765625\ldots$

$\approx 0{\cdot}0010$ (4dp)

b) To fall in dock on 12^{th} step, he must not fall in on the 10^{th} step.

So he takes 9 steps forward, then 1 step back, then next two steps forward, and he's in the dock.
Or he takes 8 steps forward, then 1 step back, then three steps forward
Or he takes 7 steps forward, then 1 step back, then four steps forward
⋮
Or he takes 1 step forward, then 1 step back, then ten steps forward
or he takes 1 step back, then eleven steps forward

10 ways

So, summarising the above, we need to consider a total of 11 steps forward, and one step back, and this is done in 10 different ways (as listed above)

∴ P(fall in dock on 12^{th} step) $= 10 \times \underbrace{(0{\cdot}5)^{11}}_{\text{forwards}} \times \underbrace{(0{\cdot}5)^{1}}_{\text{backwards}}$

$= 10 \times (0{\cdot}5)^{12}$

$= 0{\cdot}00244140625\ldots$

$= 0{\cdot}0024$ (4dp)

P(further from dock after 10 steps) = P(walked backwords more than walked forwards)

let X = number of steps walked backwards, in 10 strides

$X \sim B(10, 0{\cdot}5)$

so P(walked backwords more than walked forwards)

$= P(X > 5)$

$= P(X \geq 6)$

$= 0{\cdot}376953\ldots$ by binomCdf(10, 0·5, 6, 10)

$\approx 0{\cdot}3770$ (4dp)

Ex5B no. 9

Coin tossed 10 times

X = no. of heads in 10 tosses

$X \sim B(10, \frac{1}{2})$

$P(X \leq 4) = 0.376953125...$ by binomCdf(10, 0.5, 4)

$= \underline{\underline{0.3770}}$ (4dp)

Ex 5B no. 10.

Don

X = no. of hits

$X \sim B(10, 0.2)$

$P(X \geq 3) = 0.3222$

(binomcdf(10, 0.2, 3, 10))

Yvette.

Y = no. of hits

$Y \sim B(5, 0.4)$

$P(Y \geq 3) = 0.31744$

(binomcdf(5, 0.4, 3, 5))

So as $0.3222 > 0.31744$, Don is more likely to hit it at least 3 times.