CIMT Statistics p201 Ex10.5

1. original $\mu = 110$ mg

original $\sigma = 13$ mg

we have sample of size 19

let X = vitamin C content of one new orange

70	6
80	5 7 8 9
90	1 2 3 4 7
100	1 5 6 9 9
110	4 5 7
120	
130	6

90|3 = 93mg.

we shall assume that X is normally distributed (the stem & leaf diagram supports this assumption due to its shape)

$X \sim N(\mu, 13^2)$

$H_0 : \mu = 110$

$H_1 : \mu \neq 110$

Assume H_0 to be true, $\alpha = 5\%$ two tailed test

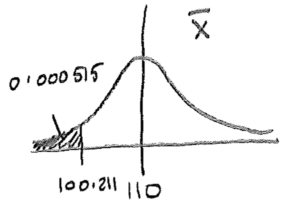
so $X \sim N(110, 13^2)$

let $\bar{X}$ = mean vitamin C content of 19 new oranges

$\bar{X} \sim N\left(110, \frac{13^2}{19}\right)$

sample mean, $\bar{x} = \frac{1904}{19} = 100.211$

$P(\bar{X} < 100.211) = P\left(Z < \frac{100.211 - 110}{\sqrt{\frac{13^2}{19}}}\right)$



$= P(Z < -3.28241)$

$= 0.000515$

p-value $= 2 \times 0.000515$

$= 0.001030$

$\ll 0.05$

So we have strong evidence to reject H_0 and we conclude that the mean vitamin C content of the new oranges is not equal to that of the original variety of oranges.

we conjecture that the new oranges have a lower mean vitamin C content

2. old engine : $\mu = 19.5$

$\sigma = 5.2$

sample size, $n = 15$

$\bar{x} = 21.6$

let X = fuel consumption of one new engine

H_0 : new engine same as old engine i.e. $\mu = 19.5$

H_1 : new engine has better fuel consumption $\Rightarrow \mu > 19.5$

Assume H_0 to be true.

$\alpha = 5\%$ one-tail test.

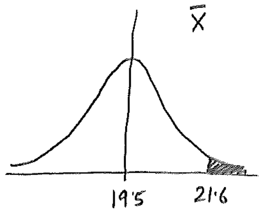
we have old engine parameters - we assume that the new engines have the same standard deviation of fuel consumption as the old ones.

we shall also assume that X is normally distributed

so, under H_0, $X \sim N(19.5, 5.2^2)$

$\Rightarrow \bar{X} \sim N\left(19.5, \frac{5.2^2}{15}\right)$ where $\bar{X}$ = mean consumption of new engines of sample size 15.

so $P(\bar{X} > 21.6) = P\left(Z > \dfrac{21.6 - 19.5}{\sqrt{\frac{5.2^2}{15}}}\right)$



$= P(Z > 1.56409)$

$= 0.058898$

> 0.05

So we do not have evidence to reject H_0, and so the mean fuel consumption of the new engine is still 19.5 miles per gallon.

Therefore the new engines seem not to be delivering a significant improvement.

3. $n = 14$

national mean = 30 mL/kg/min

st.dev = 8·6

exercise class mean = 36 mL/kg/min

let X = oxygen consumption for members of exercise class.

$E(X) = \mu$.

$H_0: \mu = 30$ (ie. exercise class no different to National norms)

$H_1: \mu > 30$ (ie. exercise class better than National norms)

Assume H_0 to be true.

$\alpha = 5\%$ one tail test

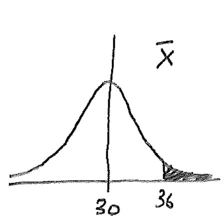
we shall assume that oxygen consumption, X, is normally distributed

we assume that X's st.dev is unchanged from National norms

So $X \sim N(30, 8{\cdot}6^2)$

$\Rightarrow \bar{X} \sim N\left(30, \frac{8{\cdot}6^2}{14}\right)$ where $\bar{X}$ = mean oxygen consumption for members of exercise class, size 14

$$P(\bar{X} > 36) = P\left(Z > \frac{36-30}{\sqrt{\frac{8{\cdot}6^2}{14}}}\right)$$



$$= P(Z > 2{\cdot}61046)$$

$$= 0{\cdot}004521$$

$$< 0{\cdot}05$$

so we have evidence to reject H_0 and conclude that the Physiotherapist's claim is justified as the mean oxygen consumption is greater than 30 mL/kg/min

4. merchant claim: mean = 50 kg
st. dev = 1 kg.

inspector weighs 60 bags giving $\bar{x} = 49.6$ kg

so let X = weight of one coal bag
let $E(X) = \mu$ and $Var(X) = 1^2$.

$H_0: \mu = 50$
$H_1: \mu < 50$

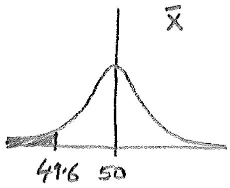
Assume H_0 to be true. $\alpha = 5\%$. one-tailed test.

we shall assume the merchant's claim about standard deviation of weight of coal bag to be true.

as $n = 60 > 20$, we can use the Central Limit Theorem to give $\bar{X}$ distributed approximately normally.

so $\bar{X} \approx N\left(50, \frac{1^2}{60}\right)$ where $\bar{X}$ = mean weight of sample of 60 bags

$$P(\bar{X} < 49.6) = P\left(Z < \frac{49.6 - 50}{\sqrt{1^2/60}}\right)$$



$$= P(Z < -3.09839)$$

$$= 0.000973$$

$$\ll 0.05$$

so we have evidence to reject H_0 and conclude that the mean weight of one coal bag is less than 50 kg. This supports the inspector's suspicions.

5. $n = 36$

pop^n mean $= \mu$ $\quad E(X) = \mu$

st. dev $= 9$ $\quad Var(X) = 9^2$

sample mean $= 47.4$

$H_0 : \mu = 50$

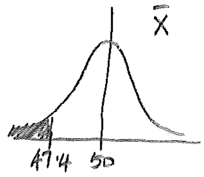
$H_1 : \mu < 50$

Assume H_0 to be true, $\alpha = 5\%$ one-tail test

as samplesize, $n = 36 > 20$, we can use Central Limit Theorem to justify $\bar{X}$ being approximately normally distributed.

as $E(X) = 50$ and $Var(X) = 9^2$, then...

$\bar{X} \approx N(50, \frac{9^2}{36})$



$P(\bar{X} < 47.4) = P\left(Z < \frac{47.4 - 50}{\sqrt{\frac{9^2}{36}}}\right)$

$= P(Z < -1.73333)$

$= 0.041518$

< 0.05

so we have evidence to reject H_0 and conclude that μ is less than 50.