1. a) $P(A) = \frac{4}{52} = \frac{1}{13}$.

b) $P(A|D) = \frac{P(A \cap D)}{P(D)}$

$= \frac{1/52}{13/52}$

$= \frac{1}{13}$.

c) $P(D) = \frac{13}{52} = \frac{1}{4}$.

d) $P(D|P) = \frac{P(D \cap P)}{P(P)}$

$= \frac{3/52}{12/52}$

$= \frac{1}{4}$.

e) $P(D|R) = \frac{P(D \cap R)}{P(R)}$

$= \frac{13/52}{26/52}$

$= \frac{1}{2}$.

f) $P(P) = \frac{12}{52} = \frac{3}{13}$.

g) $P(P|A) = \frac{P(P \cap A)}{P(A)}$

$= \frac{0}{P(A)}$

$= 0$.

h) $P(P|A') = \frac{P(P \cap A')}{P(A')}$

$= \frac{12/52}{48/52}$

$= \frac{1}{4}$.

i) $P(A|X) = \frac{P(A \cap X)}{P(X)}$

$= \frac{4/52}{50/52}$

$= \frac{2}{25}$.

j) $P(D|X') = \frac{P(D \cap X')}{P(X')}$

$= \frac{1/52}{2/52}$

$= \frac{1}{2}$.

k) $P(X|D) = \frac{P(X \cap D)}{P(D)}$

$= \frac{12/52}{13/52}$

$= \frac{12}{13}$.

l) $P(R|X) = \frac{P(R \cap X)}{P(X)}$

$= \frac{25/52}{50/52}$

$= \frac{1}{2}$.

2. a) $P(A|D) = \frac{1}{13}$

$P(A) = \frac{4}{52} = \frac{1}{13}$

$P(D) = \frac{1}{4}$

$P(A \cap D) = \frac{1}{52}$

as $P(A \cap D) = P(A)P(D)$

A & D are independent

<u>alternatively</u> : $P(A|D) = P(A)$

$\Rightarrow$ A & D are independent.

b). $\left.\begin{array}{l} P(D) = \frac{1}{4} \\ P(D|P) = \frac{1}{4} \end{array}\right\}$ D & P are independent.

c) $\left.\begin{array}{l} P(P|A) = 0 \\ P(P) = \frac{3}{13} \end{array}\right\}$ P and A are <u>not</u> independent

d) $\left.\begin{array}{l} P(R|X) = \frac{1}{2} \\ P(R) = \frac{1}{2} \end{array}\right\}$ R and X are independent

e) $\left.\begin{array}{l} P(D|R) = \frac{1}{2} \\ P(D) = \frac{1}{4} \end{array}\right\}$ D and R are <u>not</u> independent

f) $P(D) = \frac{1}{4}$

$\left.\begin{array}{l} P(D|R') = \frac{P(D \cap R')}{P(R')} \\ \quad = \frac{0}{P(R')} \\ \quad = 0 \end{array}\right\}$ D and R' are <u>not</u> independent.

EX1F cont.

3.
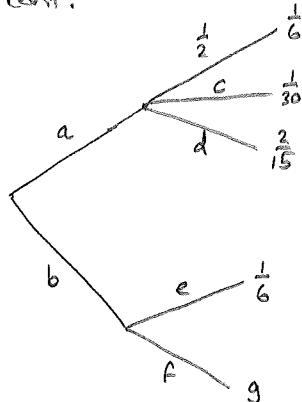
So $a+b=1$ ①

$e+f=1$ ②

$c+d=\frac{1}{2}$ ③

$\frac{1}{2}a=\frac{1}{6} \Rightarrow a=\frac{1}{3}$

$\Rightarrow b=\frac{2}{3}$ from ①

$ac=\frac{1}{30} \Rightarrow \frac{1}{3}c=\frac{1}{30}$

$\Rightarrow c=\frac{1}{10}$

$\Rightarrow d=\frac{4}{10}$ from ③.

$be=\frac{1}{6} \Rightarrow \frac{2}{3}e=\frac{1}{6}$

$\Rightarrow e=\frac{3}{12}$

$\Rightarrow e=\frac{1}{4}$

$\Rightarrow f=\frac{3}{4}$ from ②.

$bf=g \Rightarrow \frac{2}{3}\times\frac{3}{4}=g$

$g=\frac{1}{2}$.

So $\underline{\underline{a=\frac{1}{3}, b=\frac{2}{3}, c=\frac{1}{10}, d=\frac{4}{10}, e=\frac{1}{4}, f=\frac{3}{4}, g=\frac{1}{2}.}}$

Ex 1F cont.

4.   A : 8 Black
   7 white

      B : 5 Black
            7 white
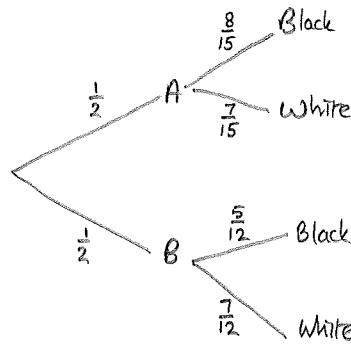
a) $P(\text{disc is white}) = P(A \text{ and white}) + P(B \text{ and white})$

$$= \tfrac{1}{2} \times \tfrac{7}{15} + \tfrac{1}{2} \times \tfrac{7}{12}$$

$$= \tfrac{7}{30} + \tfrac{7}{24}$$

$$= \underline{\underline{\tfrac{21}{40}}}.$$

b) $P(B \mid \text{white}) = \dfrac{P(B \text{ and white})}{P(\text{white})}$

$$= \dfrac{7/24}{21/40}.$$

$$= \underline{\underline{\tfrac{5}{9}}}.$$

5. 60 balls : red : blue : white
   6 : 3 : 1

   $\Rightarrow$ 36 : 18 : 6 balls.
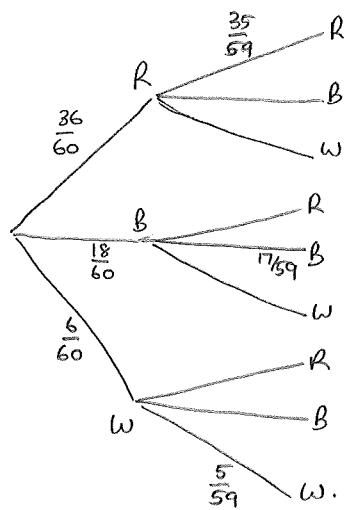
a) P(same colour) = P(RR) + P(BB) + P(WW)

$$= \frac{36}{60} \cdot \frac{35}{59} + \frac{18}{60} \cdot \frac{17}{59} + \frac{6}{60} \cdot \frac{5}{59}$$

$$= \frac{133}{295}.$$

b) P(no red ball drawn) = 1 − P(R.X) − P(XR)

$$= 1 - \frac{36}{60} - P(BR) - P(WR)$$

$$= 1 - \frac{36}{60} - \frac{18}{60} \cdot \frac{36}{59} - \frac{6}{60} \cdot \frac{36}{59}$$

$$= \frac{46}{295}.$$

c) P(WW | same colour) = $\dfrac{P(WW \text{ and same colour})}{P(\text{same colour})}$

$$= \frac{P(WW)}{P(\text{same colour})}$$

$$= \frac{\frac{6}{60} \cdot \frac{5}{59}}{133/295}$$

$$= \frac{5}{266}.$$

Ex IF cont.

6

points	1	2	3	4
P(answer correct)	$\frac{5}{6}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$

✓ = answered correctly

✗ = answered wrong

START

Total 10 points.

6 points.

4 points

Eliminated (with 3 points)

4 points.

2 points

Eliminated (1 point)

6 points.

3 points.

2 points.

Eliminated (1 point)

Eliminated (0 points)

⟵ ROUND 1 ⟶ ⟵ ROUND 2 ⟶ ⟵ ROUND 3 ⟶ ⟵ ROUND 4 ⟶
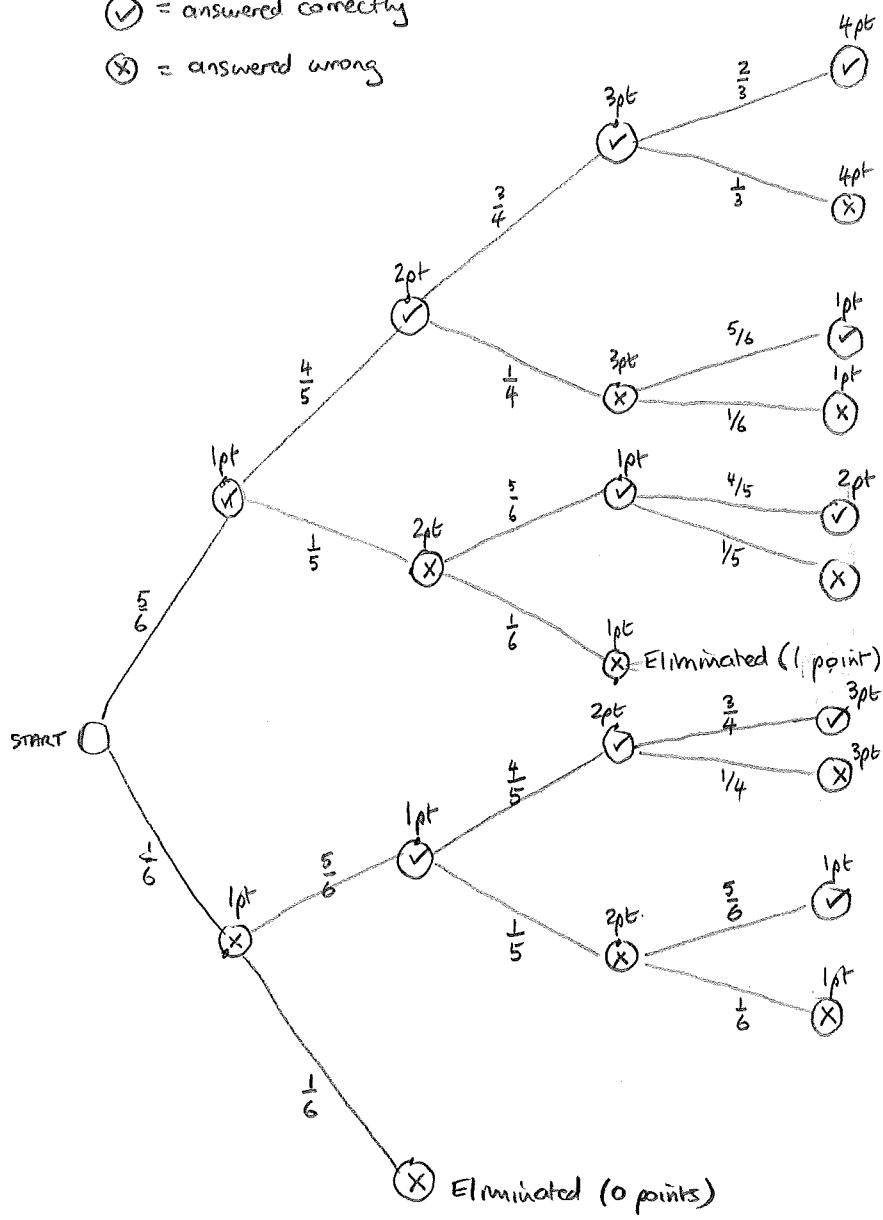
a) P(eliminated in first 4 rounds) $= \frac{5}{6} \times \frac{4}{5} \times \frac{1}{4} \times \frac{1}{6} + \frac{5}{6} \times \frac{1}{5} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6} \times \frac{1}{5} \times \frac{1}{6} + \frac{1}{6} \times \frac{1}{6}$

$\qquad\qquad\qquad\qquad = \quad \frac{1}{36} \quad + \quad \frac{1}{36} \quad + \quad \frac{1}{216} \quad + \quad \frac{1}{36}$

$\qquad\qquad\qquad\qquad = \quad \frac{19}{216}$

b) P(scored at least 6 points) $= \underbrace{\frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}}_{10 \text{ pts}} + \underbrace{\frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}}_{6 \text{ pts}} + \underbrace{\frac{1}{6} \times \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4}}_{6 \text{ pts.}}$

$\qquad\qquad\qquad\qquad = \quad \frac{2}{6} \quad + \quad \frac{1}{6} \quad + \quad \frac{3}{36}$

$\qquad\qquad\qquad\qquad = \quad \frac{7}{12}.$

EXIF cont.

7. $P(\text{fine weather}) = \frac{3}{5}$.
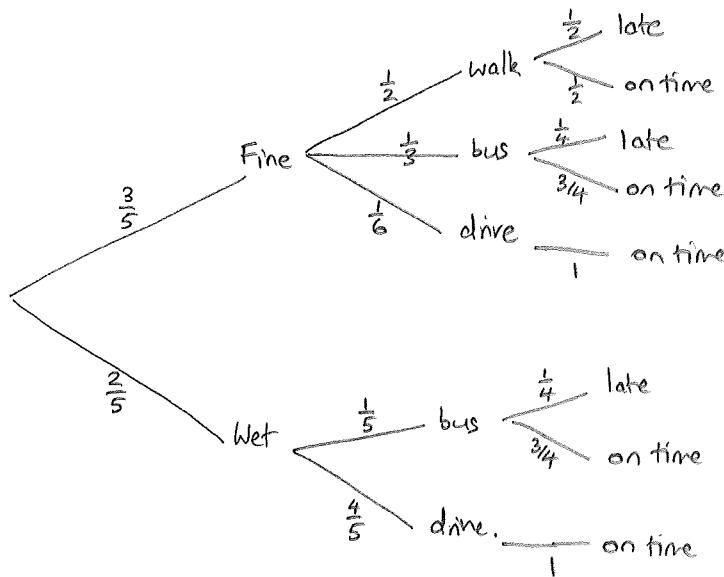
$P(\text{arrives on time}) = P(\text{Fine} \cap \text{Walk} \cap \text{OnTime}) + P(\text{Fine} \cap \text{Bus} \cap \text{OnTime}) + P(\text{Fine} \cap \text{Drive})$

$$+ P(\text{Wet} \cap \text{Bus} \cap \text{OnTime}) + P(\text{Wet} \cap \text{Drive})$$

$$= \frac{3}{5} \times \frac{1}{2} \times \frac{1}{2} + \frac{3}{5} \times \frac{1}{3} \times \frac{3}{4} + \frac{3}{5} \times \frac{1}{6} + \frac{2}{5} \times \frac{1}{5} \times \frac{3}{4} + \frac{2}{5} \times \frac{4}{5}$$

$$= \frac{3}{20} + \frac{3}{20} + \frac{1}{10} + \frac{3}{50} + \frac{8}{25}$$

$$= \frac{1}{100}(15 + 15 + 10 + 6 + 32)$$

$$= \frac{78}{100}$$

$$= \frac{39}{50}.$$

$P(\text{travels by bus} \cap \text{late}) = P(\text{Fine} \cap \text{Bus} \cap \text{Late}) + P(\text{Wet} \cap \text{Bus} \cap \text{Late})$

$$= \frac{3}{5} \times \frac{1}{3} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{5} \times \frac{1}{4}$$

$$= \frac{1}{20} + \frac{1}{50}$$

$$= \frac{7}{100}$$

$P(\text{Fine} \mid \text{Late}) = \dfrac{P(\text{Fine} \cap \text{Late})}{P(\text{Late})}$

$$= \frac{\frac{3}{5} \times \frac{1}{2} \times \frac{1}{2} + \frac{3}{5} \times \frac{1}{3} \times \frac{1}{4}}{1 - P(\text{on time})}$$

$$= \frac{\frac{3}{20} + \frac{1}{20}}{11/50}$$

$$= \frac{10}{11}$$