

$Y$  has pdf  $P(Y=y) = \frac{5-y}{10}$   $y=1,2,3,4$

a)  $E(Y) = \sum y P(Y=y)$

$$= 1 \times \frac{5-1}{10} + 2 \times \frac{5-2}{10} + 3 \times \frac{5-3}{10} + 4 \times \frac{5-4}{10}$$

$$= 1 \times \frac{4}{10} + 2 \times \frac{3}{10} + 3 \times \frac{2}{10} + 4 \times \frac{1}{10}$$

$$= \frac{20}{10}$$

$$= \underline{\underline{2}}$$

b)  $\text{Var}(Y) = E(Y^2) - E^2(Y)$

$$E(Y^2) = \sum y^2 P(Y=y)$$

$$= 1 \times \frac{4}{10} + 2^2 \times \frac{3}{10} + 3^2 \times \frac{2}{10} + 4^2 \times \frac{1}{10}$$

$$= \frac{50}{10}$$

$$= 5$$

$$\therefore \text{Var}(Y) = 5 - 2^2$$

$$= 5 - 4$$

$$= \underline{\underline{1}}$$

$$(\therefore \sigma_Y = 1)$$

Ex4C no. 2

$S$  = score on fair, 10 sided spinner

a)  $P(S=s) = \frac{1}{10} \quad s=1, \dots, 10$

b)  $E(S) = \underline{\underline{5.5}}$  by symmetry of distribution.

c)  $E(S^2) = \sum_{s=1}^{10} s^2 P(S=s)$   
 $= \sum_{s=1}^{10} s^2 \times \frac{1}{10}$   
 $= \frac{1}{10} \times (1^2 + 2^2 + 3^2 + \dots + 8^2 + 9^2 + 10^2)$   
 $= \frac{1}{10} \times 385$   
 $= \frac{77}{2}$

so  $\text{Var}(S) = E(S^2) - E^2(S)$   
 $= \frac{77}{2} - (5.5)^2$   
 $= \frac{33}{4}$

so  $\sigma_S = \sqrt{\frac{33}{4}}$

$\hat{=} 2.8723$  (4dp)

Ex4C no. 3.

$x$	0	1	2	3
$P(X=x)$	0.4	0.3	0.2	0.1

or  $P(X=x) = \frac{4-x}{10} \quad x=0,1,2,3$

a)  $E(X) = \sum x P(X=x)$

$E(X) = 1$

$E(X^2) = 2$

so  $\text{Var}(X) = E(X^2) - E^2(X)$

$= 2 - 1^2$

$= 1.$

b)  $Y = X^2 - 2X$

$x$	0	1	2	3
$y$	0	-1	0	3
$P(Y=y)$	0.4	0.3	0.2	0.1

so  $E(Y) = \sum y \cdot P(Y=y)$

$= -0.3 + 3 \times 0.1$

$= 0.$

$E(Y^2) = \sum y^2 P(Y=y)$

$= (-1)^2 \times 0.3 + 3^2 \times 0.1$

$= 0.3 + 0.9$

$= 1.2$

$\text{Var}(Y) = E(Y^2) - E^2(Y)$

$= 1.2 - 0$

$= 1.2$

Ex4C no.4.

$X$  = total score thrown

a)

	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

$x$	2	3	4	5	6
$P(X=x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

$$b) P(X > 4) = P(X=5) + P(X=6)$$

$$= \frac{21}{36}$$

$$= \underline{\underline{\frac{7}{12}}}$$

$$c) E(X) = \sum x P(X=x)$$

$$= \underline{\underline{\frac{14}{3}}}$$

$$E(X^2) = \frac{206}{9}$$

$$\text{Var}(X) = E(X^2) - E^2(X)$$

$$= \frac{206}{9} - \left(\frac{14}{3}\right)^2$$

$$= \underline{\underline{\frac{10}{9}}}$$