CIMT Statistics p199 Ex 10C

1. H_0 is an assertion that a parameter in a statistical model takes a particular value.

H_1 expresses the way in which the value of a parameter may deviate from that specified in H_0

The level of significance is chosen so that it is very unlikely that under H_0 being assumed to be true, that the result obtained is better explained by H_1

$n = 100$

$\bar{x} = 8.36g$

$s = 0.72g$

let X = weight of bar of chocolate

$E(X) = \mu \quad Var(X) = \sigma^2$

$H_0 : \mu = 8.50$

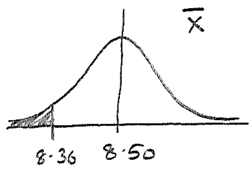
$H_1 : \mu < 8.50$

Assume H_0 to be true. $\alpha = 5\%$. One tail test

as n is large, by CLT $\bar{X} \approx N(\mu, \frac{\sigma^2}{n})$ $\quad \bar{X}$ = mean weight of n bars.

we shall estimate σ with s, as n is so large

so $\bar{X} \approx N(8.50, \frac{0.72^2}{100})$



$$p\text{-value} = P(\bar{X} < 8.36)$$

$$= P\left(Z < \frac{8.36 - 8.50}{\sqrt{\frac{0.72^2}{100}}}\right)$$

$$= P(Z < -1.9444\ldots)$$

$$= 0.025921$$

$$< 0.05$$

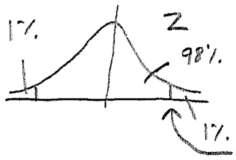
As p-value is < alpha value, we are in critical region and we have evidence to reject H_0 and conclude that the shopkeeper is justified in their complaint that the chocolate bars are less than 8.5g (but these must be very small bars of chocolate!)

A 98% CI is $\quad 8.36 \pm 2.32635\sqrt{\frac{0.72^2}{100}}$

$= 8.36 \pm 0.167497$

$= (8.1925, 8.5275)$

$\approx (8.19, 8.53)$ to 2 dp.



inv Norm$(0.99) = 2.32635$

2.

$n = 250$

$\sum x = 11872 \qquad \Rightarrow \quad \bar{x} = \frac{1}{250}\sum x = \frac{5936}{125} = 47.488$

$\sum x^2 = 646193 \qquad$ and $\quad s^2 = \dfrac{\sum x^2 - \frac{(\sum x)^2}{250}}{249} \approx 330.986$

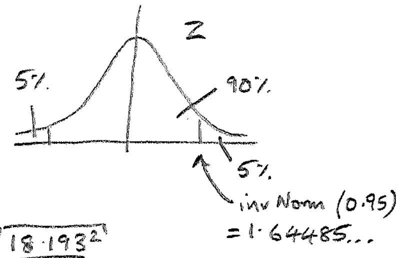
so $s = 18.193$

if X = marks scored $\quad E(X) = \mu \quad Var(X) = \sigma^2$

as n is large, by CLT then $\bar{X} \approx N(\mu, \frac{\sigma^2}{n})$

we estimate σ with s, as n is so large

so $\bar{X} \approx N(47.488, \frac{18.193^2}{250})$



Hence 90% CI is $47.488 \pm 1.64485\sqrt{\dfrac{18.193^2}{250}}$

$= 47.488 \pm 1.89261$

$= (45.5954, 49.3806)$

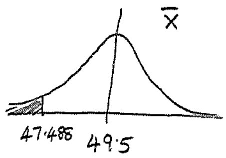
$\approx \underline{\underline{(45.6, 49.4)}} \quad (1dp)$

$H_0: \mu = 49.5$

$H_1: \mu < 49.5$

Assume H_0 to be true.

$\bar{X} \approx N(49.5, \frac{18.193^2}{250})$



$P(\bar{X} < 47.488) = P\left(Z < \dfrac{47.488 - 49.5}{\sqrt{\frac{18.193^2}{250}}}\right)$

$= P(Z < -1.74861)$

$= 0.040179$

So, from the sample taken, we would reject H_0 if the alpha value was $0.040179 \simeq 4\%$ or less

Hence set of values for α are $\alpha < 4\%$

We are given $E(X) = 45.292$ and $Var(X) = 18.761^2$

As n is large, by CLT $\bar{X} \approx N(45.292, \frac{18.761^2}{250})$

so $P(\bar{X} \geq 47.488) = P\left(Z > \dfrac{47.488 - 45.292}{\sqrt{\frac{18.761^2}{250}}}\right)$

$= P(Z > 1.85074)$

$= 0.032103...$

$\approx \underline{\underline{0.0321}} \quad (4dp)$