p145 Ex 7D no. 1

1. Mode when $f(x)$ is maximised.

a) $f(x) = \frac{3}{50}(x^2 - 4x + 5) \qquad 0 < x < 5$

$f'(x) = \frac{3}{50}(2x - 4)$

$f'(x) = 0$ when $x = 2$

$f''(x) = \frac{3}{50}(2)$

so $f''(2) > 0 \Rightarrow$ minimum at $x = 2$

so we evaluate endpoints

$f(0) = \frac{3}{50} \times 5 = \frac{3}{10}$

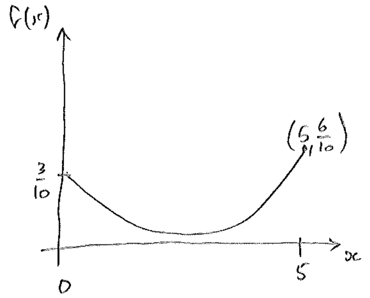
$f(5) = \frac{3}{50}(5^2 - 20 + 5)$

$= \frac{3}{50} \cdot 10$

$= \frac{3}{5}$

$= \frac{6}{10}$

so mode is when $\underline{\underline{x = 5}}$.



[answer in pdf file is not correct]

b) $f(x) = \frac{3}{13}(x^2 + 4) \qquad 0 < x < 1$

$f'(x) = \frac{3}{13}(2x)$

so $f'(x) = 0$ when $x = 0$

$f''(x) = \frac{3}{13} \cdot 2 > 0 \Rightarrow$ minimum

$\therefore$ look at end points

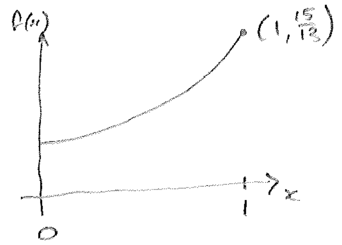
$f(0) = \frac{3}{13}(0 + 4)$

$= \frac{12}{13}$.

$f(1) = \frac{3}{13}(1^2 + 4)$

$= \frac{15}{13}$

so mode is when $\underline{\underline{x = 1}}$



Ex7D no. 2.

a) median when $\int_{-\infty}^{m} f(x)dx = \frac{1}{2}$

$f(x) = \frac{1}{8}(4-x) \quad 0<x<4$

so $\int_{0}^{m} f(x)\,dx = \frac{1}{2}$

$\int_{0}^{m} \frac{1}{8}(4-x)\,.dx = \frac{1}{2}$

$\frac{1}{8}\int_{0}^{m}(4-x)dx = \frac{1}{2}$

$\int_{0}^{m}(4-x)\,dx = 4$

$\left[4x - \frac{1}{2}x^2\right]_0^m = 4$

$(4m - \frac{1}{2}m^2) - (0-0) = 4$

$4m - \frac{1}{2}m^2 - 4 = 0$

$m^2 - 8m + 8 = 0$

$m^2 - 8m + (-4)^2 - (-4)^2 + 8 = 0$

$(m-4)^2 - 16 + 8 = 0$

$(m-4)^2 = 8$

$m - 4 = \pm 2\sqrt{2}$

$m = 4 \pm 2\sqrt{2}$

we want $0<m<4$, so

median is $\underline{\underline{4-2\sqrt{2}}}$.

b)

$f(x) = e^{-x} \quad x>0$

so $\int_{0}^{m} f(x)\,dx = \frac{1}{2}$

$\int_{0}^{m} e^{-x}\,.dx = \frac{1}{2}$.

$\left[-e^{-x}\right]_0^m = \frac{1}{2}$.

$-e^{-m} + e^{-0} = \frac{1}{2}$

$-e^{-m} + 1 = \frac{1}{2}$

$e^{-m} = \frac{1}{2}$

$\ln(e^{-m}) = \ln(\frac{1}{2})$

$-m\ln(e) = \ln(2^{-1})$

$-m\,.\,1 = -\ln 2$

$m = \ln 2$

so median is $\underline{\underline{\ln(2)}}$.