CIMT Stats p 24 Ex 1·8

1.

	1	2	3	4	5	6
1	✓	✓	✓	✓	✓	✓
1	✓	✓	✓	✓	✓	✓
2	✓	✓		✓		✓
2	✓	✓		✓		✓
3	✓		✓			✓
3	✓		✓			✓

P(one score divides into the other) $= \frac{26}{36}$

$= \frac{13}{18}$.

2. Assume that Dave & Raj are runners
Assume all runners equally likely to win

a) $P(\text{Dave and Raj win}) = P(DRXXXX) + P(RDXXXX)$

$= \frac{1}{6} \cdot \frac{1}{5} + \frac{1}{6} \cdot \frac{1}{5}$

$= \frac{2}{30}$

$= \frac{1}{15}$.

b) first, $P(\text{only Dave wins}) = P(Dxxxxx) + P(xDxxxx)$

$= \frac{1}{6} \cdot \frac{4}{5} + \frac{4}{6} \cdot \frac{1}{5}$

$= \frac{8}{30}$

$= \frac{4}{15}$

so P(only Raj wins) $= \frac{4}{15}$ also.

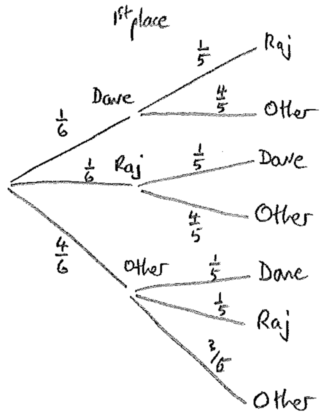
$\therefore$ P(neither wins) = 1 − P(only D wins) − P(only R wins) − P(both win)

$= 1 - \frac{4}{15} - \frac{4}{15} - \frac{1}{15}$

$= \frac{6}{15}$.

$= \frac{2}{5}$.

OR



a) $P(D)P(R|D) + P(R)P(D|J)$

$= \frac{1}{6} \times \frac{1}{5} + \frac{1}{6} \times \frac{1}{5}$

$= \frac{1}{15}$

b) $P(D)P(O|D) + P(O)P(D|O)$

$= \frac{1}{6} \times \frac{4}{5} + \frac{4}{6} \times \frac{1}{5}$.

$= \frac{4}{15}$

c) $P(O)P(O|O)$

$= \frac{4}{6} \cdot \frac{3}{5}$

$= \frac{12}{30}$

$= \frac{2}{5}$.

Ex 1.8 cont.

3. Two digit no. to be 10 up to 99 inclusive.
$\Rightarrow$ total population is 90 numbers.

a) $P(\text{divisible by } 5) = \frac{18}{90}$ $(10, 15, 20, 25, \dots, 95)$

$= \frac{1}{5}$.

b) $P(\text{divisible by } 3) = \frac{30}{90}$ $(12, 15, 18, 21, 24, 27, 30, \dots, 99)$

$= \frac{1}{3}$.

$99 - 12 = 87$
$87 \div 3 = 29$
$29 + 1 = 30$.

c) $P(\text{greater than } 50) = \frac{49}{90}$. $(51, 52, \dots, 99)$ $99 - 50 = 49$

d) $P(\text{squae number}) = \frac{6}{90}$ $(16, 25, \dots, 81)$

$= \frac{1}{15}$

Ex 1.8 cont

4. 4 coins

assume coins are fair

P(at least 3 heads from 4 tosses)

$= P(HHHT) + P(HHTH) + P(HTHH) + P(THHH) + P(HHHH)$

$= 5 \times \left(\frac{1}{2}\right)^4$

$= \frac{5}{16}.$

5.

$\xrightarrow{\text{tail}}$ $\uparrow$ head.

P(still on x-axis after 3 goes)

$= P(3 \text{ tails})$

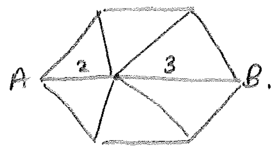
$= P(TTT)$

$= \left(\frac{1}{2}\right)^3$

$= \frac{1}{8}$.

7.

conjecture that probabilities $\propto$ areas of each segment



on TI-Nspire P(area of 1) = 0·133

P(area of 2) = 0·167

P(area of 3) = 0·2

P(area of 4) = 0·2

P(area of 5) = 0·167

P(area of 6) = 0·133

It will be interesting to see whether experimental evidence matches this algebraic conjecture.

8. A = no heads

B = at least one head

C = no tails

D = at least two tails.

a) $P(A \cap B) = 0$ is true

b) $P(A \cup B) = 1$ is true

c) $P(B \cup D) = 1$ is true

B is: HTTT HHTT HHHT HHHH

D is: HHTT HTTT TTTT

d) A' is B : HTTT HHTT HHHT HHHH

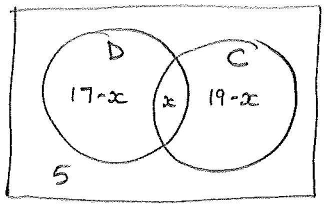
C' is at least one tail : THHH TTHH TTTH TTTT

so $P(A' \cap C') \neq 0$ so False

9. $P(dog) = \frac{17}{30}$

$P(cat) = \frac{19}{30}$

$P(neither) = \frac{5}{30}$



$\therefore\ 17 - x + x + 19 - x + 5 = 30$

$36 - x + 5 = 30$

$41 - x = 30$

$x = 11$

$\therefore\ P(\text{both cat and dog}) = \frac{11}{30}.$

OR $P(C \cap D) = P(C) + P(D) - P(C \cup D)$

$= P(C) + P(D) - (1 - P(C' \cap D'))$

$= \frac{19}{30} + \frac{17}{30} - \left(1 - \frac{5}{30}\right)$

$= \frac{36}{30} - \left(\frac{25}{30}\right)$

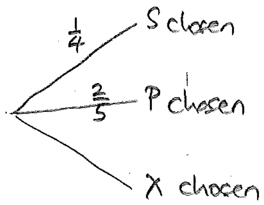
$= \frac{11}{30}.$

10. $P(\text{S chosen as goalie}) = \frac{1}{4}$

$P(\text{Paul goalie}) = \frac{2}{5}$

assume team only has one goalie

a)



$$P(\text{S or P chosen}) = \frac{1}{4} + \frac{2}{5}$$
$$= \frac{5+8}{20}$$
$$= \frac{13}{20}.$$

b) $$P(\text{neither chosen}) = 1 - P(\text{either chosen})$$
$$= 1 - \frac{13}{20}$$
$$= \frac{7}{20}.$$

11. 1478 arranged in any order (24 different orders)

A = number is odd

B = number > 4000

a) $P(A) = \frac{1}{2}$ as half of the nos. will end in 1 or 7

b) $P(B) = \frac{18}{24}$ as 16 of the nos will start with 4, 7 or 8

$= \frac{3}{4}$

c) $P(B|A) = \frac{P(B \cap A)}{P(A)}$

now, for $P(B \cap A)$ we have

4xxx ← of these 6 nos, 4 will end in 1 or 7

7xxx ← of these 6 nos, 2 will end in 1

8xxx ← of these 6 nos, 4 will end in 1 or 7

so we have 10 possible nos that meet criteria of A & B.

so $P(B \cap A) = \frac{10}{24}$

$\Rightarrow P(B|A) = \frac{10/24}{1/2}$

$= \frac{10}{12}$

$= \frac{5}{6}.$

d) $P(A \cap B) = P(B \cap A)$

$= \frac{10}{24}$ as calculated above, in part (c)

$= \frac{5}{12}.$

e) $P(A'|B) = \frac{P(A' \cap B)}{P(B)}$

now for $P(A' \cap B)$ we have

4xxx of these 6 nos, 2 will be even

7xxx of these 6 nos, 4 will be even

8xxx of these 6 nos, 2 will be even

so we have 8 possible nos. that are $A' \cap B$

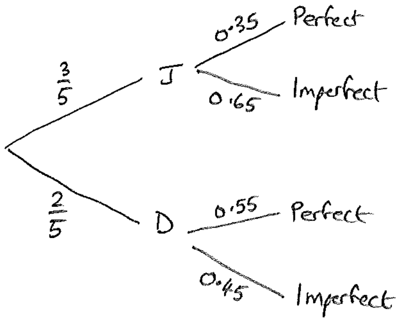
so $P(A' \cap B) = \frac{8}{24}$

$\Rightarrow P(A'|B) = \frac{8/24}{18/24}$

$= \frac{8}{18}$

$= \frac{4}{9}.$

12



a) P(job done perfectly)

$= P(J \cap \text{Perfect}) + P(D \cap \text{Perfect})$

$= \frac{3}{5} \times 0.35 + \frac{2}{5} \times 0.55$

$= \frac{3}{5} \times \frac{35}{100} + \frac{2}{5} \times \frac{55}{100}$

$= \frac{21+22}{100}$

$= \underline{\underline{\frac{43}{100}}}$

b) $P(D \mid \text{imperfect})$

$= \frac{P(D \cap \text{Imperfect})}{P(\text{Imperfect})}$

$= \frac{P(D \cap \text{Imperfect})}{1 - P(\text{perfect})}$

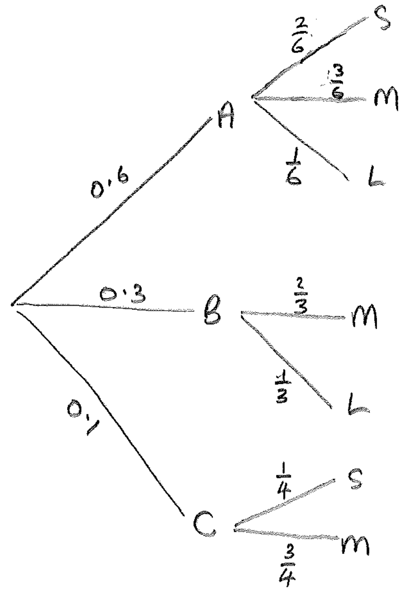
$= \frac{\frac{2}{5} \times \frac{45}{100}}{1 - \frac{43}{100}}$

$= \frac{18/100}{57/100}$

$= \frac{18}{57}$

$= \underline{\underline{\frac{6}{19}}}$.

13.



a) $P(M) = P(A \cap M) + P(B \cap M) + P(C \cap M)$

$= 0.6 \times \frac{3}{6} + 0.3 \times \frac{2}{3} + 0.1 \times \frac{3}{4}$

$= 0.3 + 0.2 + 0.075$

$= \underline{\underline{0.575}}$

b) $P(B \cap L) = 0.3 \times \frac{1}{3}$

$= \underline{\underline{0.1}}$

c) $P(C|M) = \dfrac{P(C \cap M)}{P(M)}$

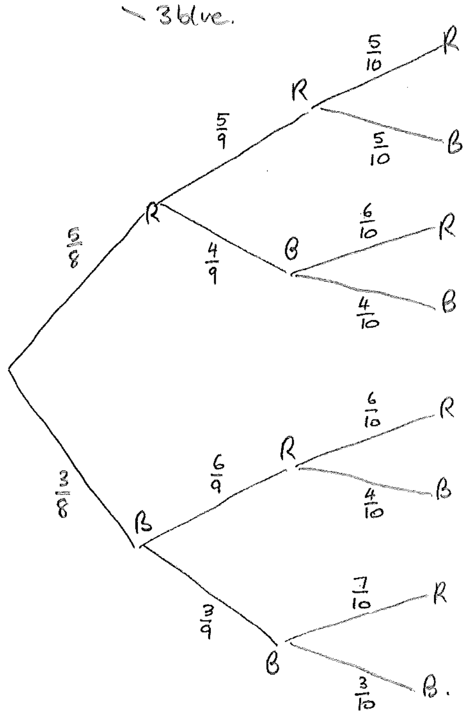
$= \dfrac{0.1 \times 3/4}{0.575}$

$= \dfrac{0.075}{0.575}$

$= \dfrac{75}{575}$ ÷ both by 25

$= \underline{\underline{\dfrac{3}{23}}}$

14 8 discs — 5 red
— 3 blue.



a) $P(3^{rd}\text{ Disc is Red}) = P(RRR)+P(RBR)+P(BRR)+P(BBR)$

$$= \frac{5}{8}\cdot\frac{5}{9}\cdot\frac{5}{10} + \frac{5}{8}\cdot\frac{4}{9}\cdot\frac{6}{10} + \frac{3}{8}\cdot\frac{6}{9}\cdot\frac{6}{10} + \frac{3}{8}\cdot\frac{3}{9}\cdot\frac{7}{10}$$

$$= \frac{125+120+108+63}{720}$$

$$= \frac{416}{720}$$

$$= \underline{\underline{\frac{26}{45}}}.$$

b) $P(\text{more Reds than Blues}) = P(RRR)+P(RRB)+P(RBR)+P(BRR)$

$$= \frac{5}{8}\cdot\frac{5}{9}\cdot\frac{5}{10} + \frac{5}{8}\cdot\frac{5}{9}\cdot\frac{5}{10} + \frac{5}{8}\cdot\frac{4}{9}\cdot\frac{6}{10} + \frac{3}{8}\cdot\frac{6}{9}\cdot\frac{6}{10}$$

$$= \frac{125+125+120+108}{720}$$

$$= \frac{478}{720}$$

$$= \underline{\underline{\frac{239}{360}}}.$$

c) $P(3^{rd}\text{ disc is Red} \mid \text{more blues than reds}) = \dfrac{P(3^{rd}\text{ disc is Red} + \text{more blues than reds})}{P(\text{more blues than reds})}$

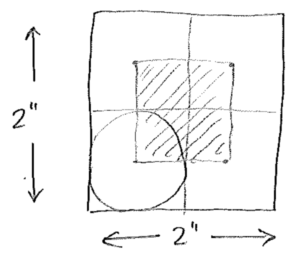
$$= \frac{P(BBR)}{1-P(\text{more reds than blues})}$$

$$= \frac{\frac{3}{8}\cdot\frac{3}{9}\cdot\frac{7}{10}}{1-\frac{478}{720}}.$$

$$= \frac{63/720}{242/720}$$

$$= \underline{\underline{\frac{63}{242}}}.$$

15.



Area of square $= 2\times 2$
$= 4 \text{ inch}^2$

Area of 'zone' for 2p's centre $= 1\times 1$
$= 1 \text{ inch}^2$

$\therefore$ P(2p lies inside square) $= \frac{1}{4}$.

in 100 goes, organiser expects 25 to lie inside square

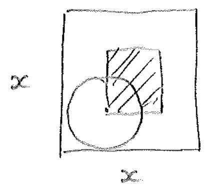
$\Rightarrow$ 75 goes has the orgairser gaining 2p each time

$\Rightarrow$ 25 goes has the orgairser losing 2p each time

$\Rightarrow$ net effect of organiser gaining 50×2p coins = £1.

So, in 100 goes the organiser would take £1.

let diameter of 10p coin to be d.
let side of square be x



we want shaded region to be $\frac{1}{3}x^2$

shaded region dimensions are $(x-d)\times(x-d)$

so $(x-d)^2 = \frac{1}{3}x^2$

$x - d = \frac{1}{\sqrt{3}}x$ (ignore -ve square root, due to context)

$x - \frac{1}{\sqrt{3}}x = d$

$x\left(1-\frac{1}{\sqrt{3}}\right) = d$

$x\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = d$

$x = \left(\frac{\sqrt{3}}{\sqrt{3}-1}\right)d$

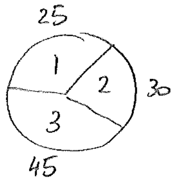
so side of square should be $\frac{\sqrt{3}}{\sqrt{3}-1}$ times larger than diameter of 10p coin.

Google states that diameter of 10p coin is 24.5 mm

$\Rightarrow$ square should be side length $\frac{\sqrt{3}}{\sqrt{3}-1}\times 24.5 = 57.9676$

≈ 58 mm (to nearest mm)

16.



if we assume frequency $\propto$ centre angles $\propto$ areas of sectors

then $25+30+45=100$

so angle sectors are

$$25 : 30 : 45$$
$$\div 5$$
$$5 : 6 : 9$$
$$\times 18$$
$$90 : 108 : 162$$

$$\underline{\underline{90^\circ, 108^\circ, 162^\circ}}$$

17 Discs numbered 1 to 20.

Split into 4 groups of 5.

Consider nos. 1 to 20 being ordered randomly into a 4×5 grid:

1	7	9	3	6
2	8	12	19	16
15	,	,	,	,
,	,	,	,	,

We consider the likelihood of 15 and 19 being in the same row (as then they will be in the same box)

Find the row with 15 in it

There are 4 other numbers in that row, with 19 possibles that can go into it

$\therefore P(15 \text{ and } 19 \text{ in the same row/box}) = \frac{4}{19}.$

If problem had been 5 groups of 4, then above model is looking for nos. in the same row

By similar logic, $P(15 \text{ and } 19 \text{ in the same column/box}) = \frac{3}{19}.$

18.

markbook left in room that he visits $\frac{1}{3}$ of time.

He visits 3 rooms in a morning

a) P(arrive a lunch having lost lunch book)

= P(left book in room 1) + P(not left in room 1, but left in room 2) + P(not left in Room 1 or Room 2, but left in Room 3)

$$= \frac{1}{3} + \frac{2}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}$$

$$= \frac{9}{27} + \frac{6}{27} + \frac{4}{27}$$

$$= \underline{\underline{\frac{19}{27}}}$$

b) P(left in 2^{nd} room | lost by lunch)

$$= \frac{P(\text{left in } 2^{nd} \text{ room and lost by lunch})}{P(\text{lost by lunch})}$$

$$= \frac{P(\text{left in } 2^{nd} \text{ room})}{P(\text{lost by lunch})}$$

$$= \frac{\frac{2}{3}\cdot\frac{1}{3}}{\frac{19}{27}}$$

$$= \frac{6/27}{19/27}$$

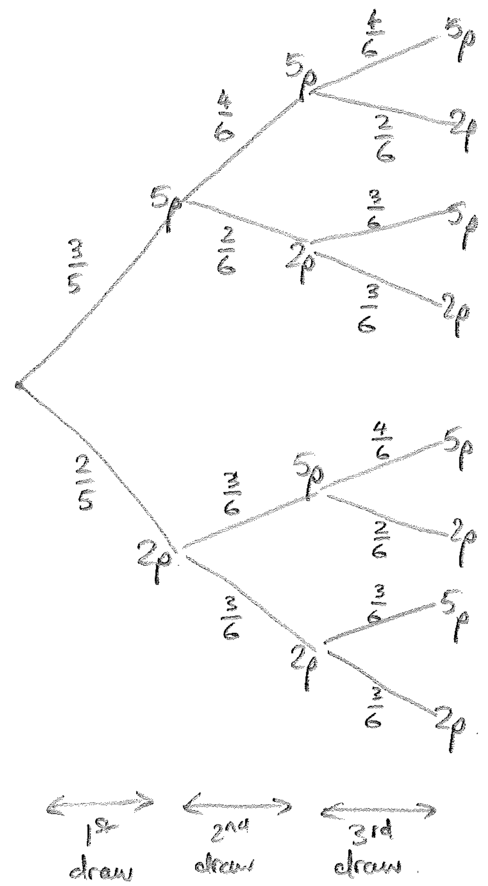
$$= \underline{\underline{\frac{6}{19}}}$$

19. A 5p 5p 5p 2p 2p

B 5p 5p 5p 2p 2p

C 5p 5p 5p 2p 2p

A into B then B into C then C drawn.



a) $P(2p \text{ selected from } C) = P(5/5/2) + P(5/2/2) + P(2/5/2) + P(2/2/2)$

$$= \frac{3}{5}\cdot\frac{4}{6}\cdot\frac{2}{6} + \frac{3}{5}\cdot\frac{2}{6}\cdot\frac{3}{6} + \frac{2}{5}\cdot\frac{3}{6}\cdot\frac{2}{6} + \frac{2}{5}\cdot\frac{3}{6}\cdot\frac{3}{6}.$$

$$= \frac{1}{180}(24 + 18 + 12 + 18)$$

$$= \frac{72}{180}$$

$$= \frac{2}{5}.$$

b) $$P(5p \text{ from } A \mid 2p \text{ from } C) = \frac{P(5p \text{ from } A \text{ and } 2p \text{ from } C)}{P(2p \text{ from } C)}$$

$$= \frac{P(5/5/2) + P(5/2/2)}{2/5}$$

$$= \frac{24/180 + 18/180}{72/180}$$

$$= \frac{42}{72}$$

$$= \frac{7}{12}.$$

19 cont.

If we had y 5p coins
x 2p coins

Conjecture $\frac{x}{x+y}$

$$P(\text{2p selected from C}) = P(5/5/2) + P(5/2/2) + P(2/5/2) + P(2/2/2)$$

$$= \left(\frac{y}{x+y} \times \frac{y+1}{x+y+1} \times \frac{x}{x+y+1}\right) + \left(\frac{y}{x+y} \times \frac{x}{x+y+1} \times \frac{x+1}{x+y+1}\right) + \left(\frac{x}{x+y} \times \frac{y}{x+y+1} \times \frac{x}{x+y+1}\right)$$

$$+ \left(\frac{x}{x+y} \times \frac{x+1}{x+y+1} \times \frac{x+1}{x+y+1}\right)$$

$$= \frac{1}{(x+y)(x+y+1)^2}\left[xy(y+1) + x(x+1)y + x^2y + x(x+1)^2\right]$$

$$= \frac{x}{(x+y)(x+y+1)^2}\left[y(y+1) + (x+1)y + xy + (x+1)^2\right]$$

$$= \frac{x}{(x+y)(x+y+1)^2}\left[y^2 + y + xy + y + xy + (x+1)^2\right]$$

$$= \frac{x}{(x+y)(x+y+1)^2}\left[y^2 + 2y + 2xy + (x+1)^2\right]$$

$$= \frac{x}{(x+y)(x+y+1)^2}\left[y^2 + 2y(x+1) + (x+1)^2\right]$$

$$= \frac{x}{(x+y)(x+y+1)^2}\left[(y + (x+1))^2\right]$$

$$= \frac{x}{(x+y)(x+y+1)^2} \cdot (x+y+1)^2$$

$$= \frac{x}{x+y}.$$

as conjectured.

20.

4 girls

$P(\text{catch ball}) = \frac{2}{3}$

a) $P(\text{not be caught by any of them})$

$= \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}.$

$= \underline{\underline{\frac{1}{81}}}$

b) $P(\text{be caught})$

$= 1 - P(\text{not caught})$

$= \underline{\underline{\frac{80}{81}}}$

21.

$P(\text{Dave gets call}) = \frac{1}{5}$

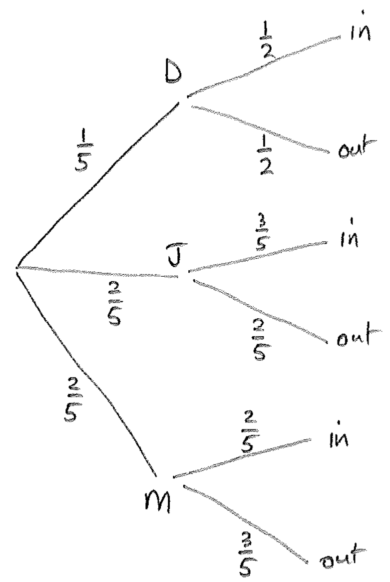
$P(\text{Jane gets call}) = \frac{2}{5}$

$P(\text{Mary gets call}) = \frac{2}{5}$

$P(\text{Dave is out}) = \frac{1}{2}$

$P(\text{Jane is out}) = \frac{2}{5}$

$P(\text{Mary is out}) = \frac{3}{5}$



a) $P(\text{call for Jean who is in}) = \frac{2}{5} \times \frac{3}{5}$

$= \frac{6}{25}$

b) $P(\text{some who is out}) = P(D \cap \text{out}) + P(J \cap \text{out}) + P(M \cap \text{out})$

$= \frac{1}{5} \cdot \frac{1}{2} + \frac{2}{5} \cdot \frac{2}{5} + \frac{2}{5} \cdot \frac{3}{5}$

$= \frac{1}{10} + \frac{4}{25} + \frac{6}{25}$

$= \frac{5 + 8 + 12}{50}$

$= \frac{1}{2}.$

c) $P(\text{call for Dave} \mid \text{some who is out}) = \dfrac{P(\text{call for Dave} \cap \text{someone who's out})}{P(\text{someone who's out})}$

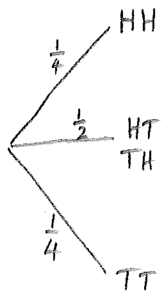
$= \dfrac{P(\text{Dave} \cap \text{out})}{P(\text{someone who's out})}$

$= \dfrac{\frac{1}{5} \times \frac{1}{2}}{\frac{1}{2}}.$

$= \frac{1}{5}.$

22.

At each toss of 2 coins:



First player will make a profit if:

HH then HT/TH $\rightarrow$ P(HH then HT/TH) $= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

HH then HH $\rightarrow$ P(HH then HH) $= \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

HT/TH then HH $\rightarrow$ P(HT/TH then HH) $= \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$\therefore$ P(player 1 makes profit) $= \frac{1}{8} + \frac{1}{16} + \frac{1}{8}$

$= \frac{5}{16}$

23. 3 minbuses.

P(minibus free) = $\frac{2}{5}$

a) P(at least one minibus free)

$= 1 - P(\text{no minibuses are free})$

$= 1 - \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= 1 - \frac{27}{125}$

$= \underline{\underline{\frac{98}{125}}}$.

b) P(all buses are free | at least one free)

$= \dfrac{P(\text{all buses are free} \cap \text{at least one free})}{P(\text{at least one free})}$

$= \dfrac{P(\text{all buses are free})}{P(\text{at least one free})}$

$= \dfrac{(\frac{2}{5})^3}{98/125}$

$= \dfrac{8/125}{98/125}$

$= \frac{8}{98}$

$= \underline{\underline{\frac{4}{49}}}$.

24.

set of 28 dominoes, all unique.

	0	1	2	3	4	5	6
0	✓	×	×	×	×	×	×
1	✓	✓	×	×	×	×	×
2	✓	✓	✓	×	×	×	×
3	✓	✓	✓	✓	×	×	×
4	✓	✓	✓	✓	✓	×	×
5	✓	✓	✓	✓	✓	✓	×
6	✓	✓	✓	✓	✓	✓	✓

✓ = dominoe piece (28 ticks)

a) $P(\text{smaller number is } 2) = \frac{17}{28}$

$= \frac{1}{4}.$

b) $P(\text{double}) = \frac{7}{28}$

$= \frac{1}{4}$

c) $P(\text{contains neither 4 or 5}) = \frac{15}{28}$

25.

3 coins.

	X	Y
H H H		
H H T	✓	
H T H	✓	
T H H	✓	
T T H	✓	✓
T H T	✓	✓
H T T	✓	✓
T T T		✓

$P(X) = \frac{6}{8}$

$P(Y) = \frac{4}{8}$

$P(X \cap Y) = \frac{3}{8}$

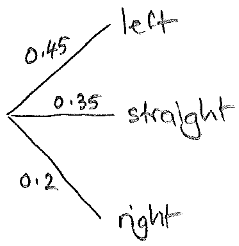
now if $P(X \cap Y) = P(X)P(Y)$ then X & Y are independent

$$\begin{aligned} LHS &= P(X \cap Y) \\ &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} RHS &= P(X)P(Y) \\ &= \frac{6}{8} \cdot \frac{4}{8} \\ &= \frac{3}{8} \\ &= LHS. \end{aligned}$$

$\therefore$ X and Y are independent.

26.



a) i) $P(\text{all go straight}) = (0.35)^3$

$= \underline{\underline{0.042875}}$

ii) $P(\text{all same direction}) = (0.45)^3 + (0.35)^3 + (0.2)^3$

$= \underline{\underline{0.142}}$

iii) $P(\text{two left and one right}) = P(LLR) + P(LRL) + P(RLL)$

$= 3 \times 0.45^2 \times 0.2$

$= \underline{\underline{0.1215}}$

iv) $P(\text{all go in different directions}) = P(LRS) \times 3!$ (for all different orders of directions)

$= 0.45 \times 0.2 \times 0.35 \times 6$

$= \underline{\underline{0.189}}$

v) $P(\text{only two turn left}) = P(LLR) \times 3 + P(LLS) \times 3$

$= 3 \times 0.45^2 \times 0.2 + 3 \times 0.45^2 \times 0.35$

$= \underline{\underline{0.334125}}$

b) $P(\text{all turn left} \mid \text{all go in same direction}) = \dfrac{P(\text{all left and all same direction})}{P(\text{all same direction})}$

$= \dfrac{P(LLL)}{P(\text{all same})}$

$= \dfrac{0.45^3}{0.142}$

$= 0.641725\ldots$

$\approx \underline{\underline{0.6417}}$ (4dp)

27.

	S	E	
W	68	62	130
G	26	32	58
B	6	6	12
	100	100	200

i) $P(\text{green estate}) = \frac{32}{200}$

$= \underline{\underline{\frac{4}{25}}}.$

ii) $P(\text{saloon}) = \frac{100}{200}$

$= \underline{\underline{\frac{1}{2}}}.$

iii) $P(\text{white} \mid \text{not saloon}) = \dfrac{P(\text{white and not saloon})}{P(\text{not saloon})}$

$= \dfrac{62/200}{100/200}$

$= \underline{\underline{\frac{31}{50}}}.$

$P(W \cup G) = \frac{130+58}{200} = \frac{188}{200} = \frac{47}{50}$

$P(S) = \frac{100}{200} = \frac{1}{2}$

$P((W \cup G) \cap S) = \frac{68+26}{200} = \frac{94}{200}$

now $P(W \cup G)P(S) = \frac{188}{200} \times \frac{1}{2} = \frac{94}{200} = P((W \cup G) \cap S)$

$\therefore$ $W \cup G$ and S are independent.

if colour & type of car are independent then $P(\text{colour} \cap \text{type}) = P(\text{colour})P(\text{type})$

pick, say, white and say, saloon so $P(W) = \frac{130}{200}$ and $P(W \cap S) = \frac{68}{200}$

$P(S) = \frac{100}{200}$

note that $\frac{130}{200} \times \frac{100}{200} \neq \frac{68}{200}$

Repeat for all other 5 categories, and none match.

Hence colour and type of car are not independent.

28.

		M	F (A)	
B	Ac.	42	28	70
C	Admin	7	13	20
	Support	26	9	35
		75	50	125

A = "female"
B = "academic"
C = "admin"

a) i) $P(A) = \frac{50}{125} = \underline{\underline{\frac{2}{5}}}$

ii) $P(A \cap B) = \underline{\underline{\frac{28}{125}}}$

iii) $P(A \cup C') = \frac{50+42+26}{125} = \underline{\underline{\frac{118}{125}}}$.

iv) $P(A' \mid C) = \frac{P(A' \cap C)}{P(C)}$

$= \frac{7/125}{20/125}$

$= \underline{\underline{\frac{7}{20}}}$.

b) i) event C is <u>not</u> independent of A :

$P(C) = 20/125$
$P(A) = 50/125$
$P(A \cap C) = 13/125 \neq P(A)P(C)$

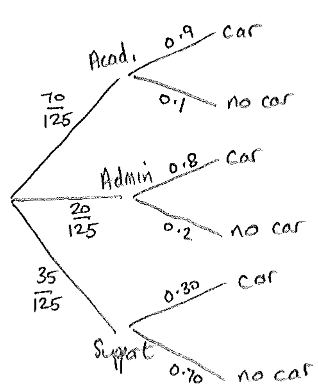
ii) event B <u>is</u> independent of A :

$P(B) = \frac{70}{125}$
$P(A) = \frac{50}{125}$
$P(A \cap B) = \frac{28}{125} = P(A)P(B)$.

iii) event A' is mutually exclusive of A

$P(A \cap A') = 0$.

c)



P (car owned) = P (Acad ∩ car) + P (admin ∩ car) + P (support ∩ car)

$= \frac{70}{125} \times 0.9 + \frac{20}{125} \times 0.8 + \frac{35}{125} \times 0.3$

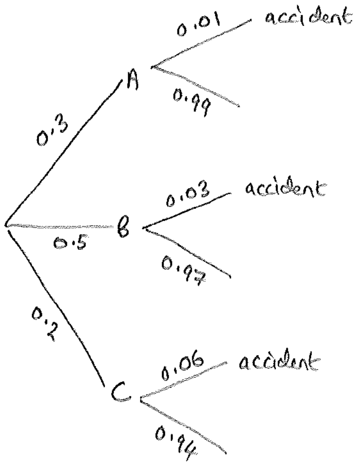
$= \underline{\underline{\frac{179}{250}}}$.

P(support | car) = $\frac{P(\text{support} \cap \text{car})}{P(\text{car})}$

$= \frac{\frac{35}{125} \times 0.3}{179/250}$

$= \underline{\underline{\frac{21}{179}}}$.

29.



a) $P(C \cap \text{accident}) = 0.2 \times 0.06$
$= \underline{\underline{0.012}}.$

b) $P(\text{accident}) = P(A \cap \text{accident}) + P(B \cap \text{accident}) + P(C \cap \text{accident})$
$= 0.3 \times 0.01 + 0.5 \times 0.03 + 0.2 \times 0.06$
$= 0.003 + 0.015 + 0.012$
$= \underline{\underline{0.03}}$

c) $$P(C \mid \text{accident}) = \frac{P(C \cap \text{accident})}{P(\text{accident})} = \frac{0.2 \times 0.06}{0.03} = \frac{0.012}{0.030} = \underline{\underline{0.4}}.$$

d) $$P(A \mid \text{no accident in 10 years}) = \frac{P(A \cap \text{no accident in 10 years})}{P(\text{no accidents in 10 years})}$$
$$= \frac{0.3 \times 0.99^{10}}{0.3 \times 0.99^{10} + 0.5 \times 0.97^{10} + 0.2 \times 0.94^{10}}$$
$$= \frac{0.271315}{0.74775}$$
$$= 0.362841\ldots$$

similarly $$P(B \mid \text{no accidents in 10 years}) = \frac{0.5 \times 0.97^{10}}{0.74775} = 0.493096\ldots$$

and $$P(C \mid \text{no accidents in 10 years}) = \frac{0.2 \times 0.94^{10}}{0.74775} = 0.144063\ldots$$

$0.362841 : 0.493096 : 0.144063$

$\div 0.3628$

$1 : 1.35898 : 0.397041$

$\times 2.71$

$2.71 : 3.68284.. : 1.07598$ (to 14dp)

$\underline{\underline{2.71 : 3.68 : 1.08}}$ (to 2dp)

30.

	% hospital	% underweight	% poor quality	other
A	55	3	7	1% both underweight & poor quality
B	35	5	12	poor quality independent of underweight
C	10	6	20	40% underweight contain poor quality.
	100			

a) P(tin from A) = 0.55.

b) P(tin underweight) = P(A ∩ underweight) + P(B ∩ underweight) + P(C ∩ underweight)

$= 0.55 \times 0.03 + 0.35 \times 0.05 + 0.10 \times 0.06$

$= 0.04$

c) P(B's underweight ∩ poor) = P(underweight)P(poor) as independent

$= 0.05 \times 0.12$

$= 0.006.$

d) P(poor quality | underweight from A) = $\dfrac{P(\text{poor} \cap \text{underweight})}{P(\text{underweight})}$

$= \dfrac{0.01}{0.03}$ ← given to us in info.

$= \dfrac{1}{3}.$

e) P(C's tin underweight and poor) = P(underweight)P(poor | underweight)

$= 0.06 \times 0.40$

$= 0.024$

f) P(underweight | C's tin poor quality) = $\dfrac{P(\text{C's tin underweight} \cap \text{poor})}{P(\text{C's tin poor})}$

$= \dfrac{0.024}{0.20}.$

$= 0.12.$

g) P(underweight and poor) = P(A ∩ underweight ∩ poor) + P(B ∩ underweight ∩ poor) + P(C ∩ underweight ∩ poor)

$= 0.55 \times 0.01 + 0.35 \times 0.05 \times 0.12 + 0.1 \times 0.06 \times 0.40$

$= 0.01$

h) P(A | poor and underweight) = $\dfrac{P(A \cap \text{underweight} \cap \text{poor})}{P(\text{underweight} \cap \text{poor})}$

$= \dfrac{0.55 \times 0.01}{0.01}$

$= 0.55.$