p168 Ex 8E no. 1

1a) $P(\text{smoke}) = 0.4$

X = no. people who smoke out of 50

$X \sim B(50, 0.4)$

$P(X>25) = 0.057344$ by binomCdf(50,0.4,26,50)

if Y is Normal approximation (as $np>5$ and $nq>5$)

$Y \sim N(50\times0.4, 50\times0.4\times0.6)$

$Y \sim N(20, 12)$

$P(X>25) \doteqdot P(Y>25.5)$ using c.c.

$= P\left(Z > \frac{25.5-20}{\sqrt{12}}\right)$

$= P(Z > 1.58771)$

$= 0.056176$ by normCdf(1.58771, 9E99)

$= \underline{0.0562}$ (4dp)

b) let X = no. people smoke out of 150

$X \sim B(150, 0.4)$

$P(X<50) = 0.038867$ by binomCdf(150,0.4,0,49)

if Y is normal approximation to X, and $np>5$ and $nq>5$

$Y \sim N(150\times0.4, 150\times0.4\times0.6)$

$Y \sim N(60, 6^2)$

$P(X<50) \doteqdot P(Y<49.5)$ by c.c.

$= P\left(Z < \frac{49.5-60}{6}\right)$

$= P(Z < -1.75)$

$= 0.040059$ by normCdf(-9E99, -1.75)

$= \underline{0.0401}$ (4dp)

Ex 8E no. 2

X = no. of Story Party voters

$X \sim B(200, 0.32)$

$P(X > 80) = P(X \geq 81)$
$= 0.006928$ from binomCdf(200, 0.32, 81, 200)

if Y is normal approx. to X as $np > 5$ and $nq > 5$

$Y \sim N(200 \times 0.32, 200 \times 0.32 \times 0.68)$

$Y \sim N(64, 43.52)$

so $P(X \geq 81) \doteqdot P(Y > 80.5)$ by c.c.

$= P\left(Z > \dfrac{80.5 - 64}{\sqrt{43.52}}\right)$

$= P(Z > 2.50115)$

$= 0.00619$

$= \underline{0.0062}$ (4dp)

Ex 8E no. 3

$P(\text{defect}) = 0.05$

X = no. defective items in a day's production

$X \sim B(500, 0.05)$

now $E(X) = 500 \times 0.05$
$= 25$

So, as 40 is above 25, we want to know what is $P(X \geq 40)$?

exactly, $P(X \geq 40) = 0.002701$ from binomcdf(500, 0.05, 40, 500)

approximate with Normal

if Y is approx to X, as $np > 5$ and $nq > 5$

then $Y \sim N(25, 23.75)$

so $P(X \geq 40) \doteq P(Y \geq 39.5)$ by c.c.

$= P\left(Z \geq \frac{39.5 - 25}{\sqrt{23.75}}\right)$

$= P(Z \geq 2.97534)$

$= 0.001463$

from normCdf(2.975, 9E99)

approximate with Poisson

if Y is approx to X, as n is large and p is small,

then $Y \sim Po(500 \times 0.05)$

$\Rightarrow Y \sim Po(25)$

So $P(X \geq 40) \doteq P(Y \geq 40)$

$= 1 - P(Y \leq 39)$

$= 1 - 0.996556$ from poissCdf(25, 0, 39)

$= 0.0034444$

so, whether you use the exact calculation, or either of the acceptable approximations, the probability of gaining 40 defects, or more, is less than 1% significance level.

This places the '40' result in the most extreme 1% tail of the distribution

∴ 40 defects out of a production line of 500, is *not* an expected result.

Ex 8E no. 4.

X = no. tickets sold per day

$X \sim Po(30)$

a) $P(X<20) = P(X \leq 19)$

$= 0.021873$ from poissCdf (30,0,19)

if Y is approx to Normal, then $Y \sim N(30, 30)$ as $\lambda > 10$

$$P(X \leq 19) \doteqdot P(Y < 19.5) \text{ by c.c.}$$

$$= P\left(Z < \frac{19.5-30}{\sqrt{30}}\right)$$

$$= P(Z < -1.91703)$$

$$= 0.027617 \quad \text{from normCdf } (-9E99, -1.91703)$$

$$\underline{\underline{= 0.0276}} \ (4dp)$$

b) let W = demand for tickets, per week.

note slight change in phrasing of random variable definition

so $W \sim Po(5 \times 30)$

$W \sim Po(150)$

$P(\text{all 180 tickets sold}) = P(\text{demand is} \geq 180)$

$= P(W \geq 180)$

$= 1 - P(W \leq 179)$

$= 1 - 0.990582$ from poissCdf (150, 0, 179)

$= 0.009418$

if V is approx to Normal, then $V \sim N(150, 150)$ as $\lambda > 10$

$$P(W \geq 180) \doteqdot P(V > 179.5) \text{ by c.c.}$$

$$= P\left(Z > \frac{179.5-150}{\sqrt{150}}\right)$$

$$= P(Z > 2.40866)$$

$$= 0.008005$$

$$\underline{\underline{= 0.0080}} \ (4dp)$$

Ex 8E no. 5.

$X =$ no. parts demanded per week

$X \sim Po\ (20)$

let no. parts stocked $= n$.

$P(\text{run out of parts}) = P(X \geqslant n)$

we want this to be $\frac{1}{20} = 0.05$

so $P(X \geqslant n) = 0.05$.

solving exactly: nSolve(poissCdf(20, n+1, 9E99) = 0.05, n)

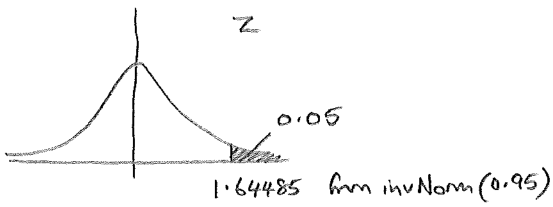
gives an error ☹.

let Y be approx to X, as $\lambda > 10$

$Y \sim N(20, 20)$

so $P(X \geqslant n) \doteqdot P(Y > n - \frac{1}{2})$ by c.c.

$= P\left(Z > \frac{(n-\frac{1}{2}) - 20}{\sqrt{20}}\right)$



so $\frac{(n-\frac{1}{2}) - 20}{\sqrt{20}} = 1.64485$

$n - 20\frac{1}{2} = 1.64485 \times \sqrt{20}$

$n = 20.5 + 1.64485 \times \sqrt{20}$

$n = 27.856$

if $n = 27$, then $P(X \geqslant n) > 0.05$ (ie. slightly more than 1 in 20)

if $n = 28$, then $P(X \geqslant n) < 0.05$ (ie. slightly less than 1 in 20)

So the shop should stock 27 or 28 parts depending on whether it wants to average slightly more, or less, than 1 in 20 for when it is out of stock of those parts.