CIMT Statistics p129 Ex 6·4

1. let $G_S \sim Po(2{\cdot}5)$ $\qquad P(G_S = g) = \dfrac{2{\cdot}5^g e^{-2{\cdot}5}}{g!}$

$G_P \sim Po(1{\cdot}5)$

a) $P(G_S \geq 2) = 1 - P(G_S \leq 1)$

$= 1 - 0{\cdot}287297\ldots$ from poisscdf $(2{\cdot}5, 0, 1)$

$= \underline{\underline{0{\cdot}7127}}$ (4dp)

b) P (at most 3 goals in total)

= P(no goals) + P(one goal) + P(two goals) + P(three goals)

$= P(G_S=0, G_P=0) + P(G_S=0, G_P=1) + P(G_S=1, G_P=0) + P(G_S=0, G_P=2)$
$P(G_S=1, G_P=1)$
$P(G_S=2, G_P=0)$
$+ P(G_S=0, G_P=3)$
$P(G_S=1, G_P=2)$
$P(G_S=2, G_P=1)$
$P(G_S=3, G_P=0)$

$= 0{\cdot}08 \times 0{\cdot}22 + 0{\cdot}08 \times 0{\cdot}33 + 0{\cdot}21 \times 0{\cdot}22 + 0{\cdot}08 \times 0{\cdot}25 + 0{\cdot}21 \times 0{\cdot}33 + 0{\cdot}26 \times 0{\cdot}22 +$
$0{\cdot}08 \times 0{\cdot}13 + 0{\cdot}21 \times 0{\cdot}25 + 0{\cdot}26 \times 0{\cdot}33 + 0{\cdot}21 \times 0{\cdot}22$

$= 0{\cdot}018 + 0{\cdot}027 + 0{\cdot}046 + 0{\cdot}021 + 0{\cdot}069 + 0{\cdot}057 + 0{\cdot}010 + 0{\cdot}052 + 0{\cdot}086 + 0{\cdot}048$

$= 0{\cdot}43347012\ldots$

$\approx \underline{\underline{0{\cdot}4335}}$ (4dp)

2.

X = no. people absent

$X \sim B(160, 0.08)$

$P(X \geq 5) = 0.996697\ldots$ exactly, using binomCdf $(160, 0.08, 5, 160)$

if approx X with $Y \sim Po(160 \times 0.08)$

$Y \sim Po(12.8)$

so $P(X \geq 5) \approx P(Y \geq 5)$

$= 1 - P(Y \leq 4)$

$= 1 - 0.004317\ldots$ from poisscdf $(12.8, 0, 4)$

$= 0.995683\ldots$

$\approx \underline{\underline{0.9957}}$ (4dp)

3

$X \sim Po\ (3{\cdot}99)$

most likely value is when $P(X=x)$ is at its highest

This will either be $P(X=3)$; $P(X=4)$ or $P(X=5)$ as the mode is near the expected value in a Poisson distribution.

$P(X=3) = 0{\cdot}195854...$

$P(X=4) = 0{\cdot}195364...$

$P(X=5) = 0{\cdot}155901...$

so most likely value will be 3 that's expected to happen 19·59% of the time, compared to 4 that happens only 19·54% of the time.

4. Van Hire firm has 12 vehicles

X = no. vehicles wanting hired per day ← assumed time scale.

$X \sim Po(9.5)$

a) demand will exceed supply when $X > 12$

so $P(X \geq 13) = 1 - P(X \leq 12)$

$= 1 - 0.83643$ from poissCdf(9.5,0,12)

$= 0.16357$

∴ out of 25 days, we'd expect demand to exceed supply on 25×0.16357

≈ 4.08926

$\approx$ 4 days

b) $P(X=0) = 0.000075$

so all vehicles idle for 25×0.000075

≈ 0.001871

$\approx$ 0 days.

c) 3 vans can be serviced if $X \leq 9$

so $P(X \leq 9) = 0.521826\ldots$ from poissCdf(9.5,0,9)

∴ out of 25 days, you could service 3 vans for $25 \times 0.521826\ldots$

≈ 13.0457

$\approx$ 13 days

7. let $X \sim Po(5)$

$P(X=3) = 0.140374$ from poissPdf $(5,3)$

let Y = no. of threes

$Y \sim B(4, 0.140374)$

$P(Y=2) = 0.087366$ from binomPdf$(4, 0.140374, 2)$

≈ 0.0874 (4dp)

8. X = no. copies magazine sold per week

$X \sim Po(10)$

$P(X<4) = P(X \leq 3)$

$= 0.01034$ from poissCdf $(10,0,3)$

≈ 0.0103 (4dp)

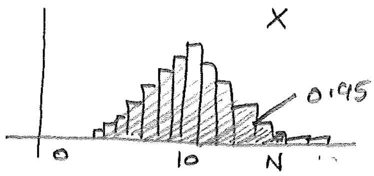
let him stock N at the start of week
if more than N are wanted, then he can't deliver
we want $P(X>N) < 0.05$

$1 - P(X \leq N) < 0.05$

$1 < 0.05 + P(X \leq N)$

$0.95 < P(X \leq N)$

$P(X \leq N) > 0.95$



so $P(X \leq 15) = 0.95126$

$P(X \leq 14) = 0.916542$ by drawing graph, and table, of $f(x) = \text{poissCdf}(10,0,x)$

$\therefore$ (by trial and error), he needs to stock 15 copies, so that the chances that he cannot provide a customer with a copy is less than 5%

13. Open 24 hours a day.

Poisson distribution assumes that arrival of customers

- are independent of one another
- have constant rate of arrival

It is unlikely that arrivals will be independent, as people will arrive at the service station in groups, by car or coach load.

Also, arrivals rate at, say, 2am, will be different to say, 10am.

Hence - if it poisson - the rates of arrival would be different according to the times of the day.

15. 8 places available.

after 1 month, if ≤ 3 places taken, course is cancelled.

if ≥ 4 places taken, course runs

let X = no. applicants per month

$X \sim Po(3.6)$

a) $P(\text{course cancelled}) = P(X \leq 3)$

$= 0.515216\ldots$ from poissCdf(3.6,0,3)

$\approx \underline{0.5152}$ (4dp)

b) $P(\text{course full}) = P(X \geq 8)$

$= 1 - P(X \leq 7)$

$= 1 - 0.969211\ldots$ from poissCdf(3.6,0,7)

$= 0.030789\ldots$

$\approx \underline{0.0308}$ (4dp)

c) $P(\text{place available at start of } 2^{nd} \text{ month}) = P(4 \leq X \leq 7)$

$= 0.453995\ldots$ from poissCdf(3.6,4,7)

$\approx \underline{0.4540}$ (4dp)

d) $P(\text{course runs with 8 students}) = P(X=4)P(X\geq 4) + P(X=5)P(X\geq 3) + P(X=6)P(X\geq 2) + P(X=7)P(X\geq 1) + P(X=8)P(X\geq 0)$

$= 0.19 \times 0.48 + 0.14 \times 0.70 + 0.08 \times 0.87 + 0.04 \times 0.973 + 0.02 \times 1$

$= 0.321366$

$\approx \underline{0.3214}$ (4dp)

(disagree with printed answers of 0.333)

e) $P(\text{cancelled}) = 0.515216\ldots$

$P(\text{run}) = 1 - P(\text{cancelled}) = 0.484784\ldots$

$P(\text{run, cancel, cancel, cancel}) = {}^4C_1 \times 0.4848 \times 0.5152^3$

$= 0.265202\ldots$

$\approx \underline{0.2652}$ (4dp)

f) $P(\text{run with 8 students at least twice})$

$= P(\text{run} \times 2) + P(\text{run} \times 3) + P(\text{run} \times 4)$

$= {}^4C_2 (0.3214)^2 (0.5152)^2 + {}^4C_3 (0.3214)^3 (0.5152) + {}^4C_4 (0.3214)^4$

$= 0.386138\ldots$

$\approx \underline{0.3861}$ (4dp)