CIMT Stats p5 Ex 1A.

1. $P(\text{pick even}) = \frac{4}{8}$

$= \frac{1}{2}$.

2.

D1 \ D2	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

$P(\text{product} > 6) = \frac{22}{36}$

$= \frac{11}{18}$.

3. 10p 10p 10p 50p 50p

	10p	10p	10p	50p	50p
10p	×	20	20	60	60
10p	20	×	20	60	60
10p	20	20	×	60	60
50p	60	60	60	×	100
50p	60	60	60	100	×

$P(\text{sum to } 60) = \frac{12}{20}$

$= \frac{3}{5}$.

4. P(both born on same weekday)

= P(2nd person born on same weekday as 1st person)

$= \frac{1}{7}$.

5. HTTTTT
THTTTT
TTHTTT
TTTHTT
TTTTHT
TTTTTH

Ex 1A cont.

6. Assume two unbiased D6.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	(18)
4	4	8	12	(16)	(20)	(24)
5	5	10	15	(20)	(25)	(30)
6	6	12	(18)	(24)	(30)	(36)

Top 11 scores are ≥ 16

$\therefore P(\text{score} \geq N) = \frac{11}{36}$

$\Rightarrow N = 16.$

7. Pierre highest $\Rightarrow$ wins 7p.
Pierre not highest $\Rightarrow$ Pierre loses 5p.

Assume fair D6.

Pierre \ Julian	1	2	3	4	5	6
1	-5	-5	-5	-5	-5	-5
2	7	-5	-5	-5	-5	-5
3	7	7	-5	-5	-5	-5
4	7	7	7	-5	-5	-5
5	7	7	7	7	-5	-5
6	7	7	7	7	7	-5

$P(\text{Pierre scores highest}) = \frac{15}{36}$

$P(\text{Pierre not highest}) = \frac{21}{36}.$

Hence ratio of highest : non highest

15 : 21

5 : 7

Hence game is fair as the winning amount compensates for the reduced/enlarged chance of that event happening.

if Pierre's 2 becomes 5...

Pierre \ Julian	1	2	3	4	5	6
1	-5	-5	-5	-5	-5	-5
5	7	7	7	7	-5	-5
3	7	7	-5	-5	-5	-5
4	7	7	7	-5	-5	-5
5	7	7	7	7	-5	-5
6	7	7	7	7	7	-5

now $P(\text{Pierre scores highest}) = \frac{18}{36}$

$P(\text{Pierre not highest}) = \frac{18}{36}$

$\therefore$ Pierre's winning probability has increased from $\frac{15}{36}$ to $\frac{18}{36}$.

His previous expected winnings were:

$$7 \times \frac{15}{36} - 5 \times \frac{21}{36}$$

$$= 7 \times \frac{5}{12} - 5 \times \frac{7}{12}$$

$$= 0.$$

His new expected winnings are:

$$7 \times \frac{18}{36} - 5 \times \frac{18}{36}$$

$$= 2 \times \frac{18}{36}$$

$$= 1p$$

Ex 1A cont.

8. P(at least one is a picture card)

$= 1 - P(\text{neither is a picture card})$

$= 1 - P(\text{non picture card}) \times P(\text{non picture card})$

$= 1 - \frac{40}{52} \times \frac{40}{52}$

$= 1 - \left(\frac{10}{13}\right)^2$

$= 1 - \frac{100}{169}$

$= \frac{69}{169}$.

9. P(Shanif and Rajjit sit together)

$= P(\text{Shanif sits on left or right of Rajjit})$

$= 2 \times P(\text{Shanif sits on right of Rajjit})$

$= 2 \times P(\text{Shanif sits in one of the seven remaining seats})$

$= 2 \times \frac{1}{7}$

$= \frac{2}{7}$.