p112 Ex 5.3 no. 1

1 a) X = no. of tubes that shatter

$X \sim B(25, 0.15)$

i) $P(X \geq 4) = 0.528879$ from binomCdf $(25, 0.15, 4, 25)$

$= 0.5289$ (4dp)

ii) let Y = no. tubes that survive

$Y \sim B(25, 0.85)$

$P(16 \leq Y \leq 20) = 0.315752..$ from binomCdf $(25, 0.85, 16, 20)$

$= 0.3158$ (4dp)

let T = no. tubes that survive

$T \sim B(30, p)$

$P(T = 30) = 0.4$

so $p^{30} = 0.4$

$p = \sqrt[30]{0.4}$

$p = 0.969919$

$p = 0.9699$ (4dp)

b) i) Not a binomial as it is not a fixed number of trials

ii) Yes, as it's a fixed number of trials with fixed probability of success, assuming that the monkey's abilities to press the button at the right time remains unchanged over the 20 repetitions.

If the monkey's responses improve (or decline – he may not want any more food!) then it won't be Binomial as p is changing

Ex 5.3 no. 2

$p < \frac{1}{6}$

X = no. sixes in 25 throws.

$X \sim B(25, p)$

if st. dev of $X = 1.5$

$\Rightarrow 25pq = 1.5^2$

$p(1-p) = 0.09$.

$p - p^2 = 0.09$

$0 = p^2 - p + 0.09$

$p = 0.1$ or 0.9 from polyRoots$(p^2 - p + 0.09, p)$

now, as $p < \frac{1}{6}$, $\underline{\underline{p = 0.1}}$

so $P(X=3) = 0.226497$ from binompdf$(25, p, 3)$

$\doteq \underline{\underline{0.23}}$. (2dp)

Ex 5.3 no. 3.

3 a) ① if X = no. of black marbles, say, then this is Binomial due to the marbles being replaced after each draw. If X is something else that is not counting one colour, then no.

② still a Binomial as only p will have changed, depeng upon which colour you want

③ not Binomial as p would change after each draw.

b) X = no. faulty bolts in sample of 10

$X \sim B(10, 0.2)$

i) $P(X \leq 2) = 0.6778$ from binomCdf (10, 0.2, 2)

≈ 0.68 (2sf)

ii) Y = no. non faulty bolts

$Y \sim B(10, 0.8)$

$E(Y) = 10 \times 0.8 = 8$

$Var(Y) = 10 \times 0.8 \times 0.2 = 1.6$.

Ex 5.3 no. 4

a) $X =$ no. completed crosswords in a week of 6 days.

$X \sim B(6, \frac{8}{10})$

$E(X) = 6 \times \frac{8}{10} = \underline{\underline{4.8}}$

$\sigma_X = \sqrt{Var(X)}$

$= \sqrt{6 \times 0.8 \times 0.2}$

$= \sqrt{0.96}$

$= 0.979796$

$= \underline{\underline{0.9797 \ (4dp)}}$

b) $P(X \geq 5) = 0.65536$ by binomcdf $(6, 0.8, 5, 6)$

$= \underline{\underline{0.6554}}$ (4dp)

c) let $Y =$ no. completed crosswords in remaining 5 days (each day is independent)

$Y \sim B(5, 0.8)$

$P(Y \geq 4) = 0.73728$ by binomCdf $(5, 0.8, 4, 5)$

$= \underline{\underline{0.7373}}$ (4dp)

d) $P(X \leq 4) = 0.34464$

let $C =$ no. times he completes ≤ 4 crosswords in a week

$C \sim B(4, 0.34464)$

$P(C=1) = 0.38803\ldots$ from binompdf $(4, 0.34464, 1)$

$= \underline{\underline{0.3880}}$ (4dp)

Ex5.3 no.5.

i) $N=$ no. of example sheets completed by the end of the course

$N \sim B(10, \frac{2}{3})$

So

n	0	1	2	3	4	5	6	7	8	9	10
$P(N=n)$	0	0	0	0.02	0.06	0.14	0.23	0.26	0.20	0.09	0.02

lazy students!

industrious students!

$P(\text{student passed exam at end of course}) = \frac{n}{10}$

n	0	1	2	3	4	5	6	7	8	9	10
P(passed exam)	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	1.

a) $P(\text{student passes exam}) = P(\text{student did no example sheets} \cap \text{passes exam})$
$+ P(\text{did 1 example sheet} \cap \text{passes exam})$
$+ P(\text{did 2 example sheets} \cap \text{passes exam})$
$\vdots$
$+ P(\text{did all 10 example sheets} \cap \text{passes exam})$

$$= 0 \times \frac{0}{10} + 0 \times \frac{1}{10} + 0 \times \frac{2}{10} + 0.02 \times \frac{3}{10} + 0.06 \times \frac{4}{10} + \dots + 0.09 \times \frac{9}{10} + 0.02 \times \frac{1}{10}$$

$$= \sum_{n=0}^{10} P(N=n) \times \frac{n}{10}$$

$$= \sum_{n=0}^{10} \left({}^{10}C_n \left(\tfrac{2}{3}\right)^n \left(\tfrac{1}{3}\right)^{10-n} \times \frac{n}{10} \right) \quad \text{or} \quad \sum_{n=0}^{10} \left(\text{binompdf}(10, \tfrac{2}{3}, n) \cdot \frac{n}{10} \right)$$

$$= \underline{\underline{\frac{2}{3}}}$$ by typing above expressions into TI-Nspire.

b) we want $P(\text{student completed} \le 4 \text{ sheets} \mid \text{they passed the exam})$

$$= \frac{P(\text{student completes} \le 4 \text{ sheets and passes exam})}{P(\text{they pass the exam})}$$

$$= \frac{\sum_{n=0}^{4} P(N=n) \times \frac{n}{10}}{2/3}$$

$$= \frac{0.028282}{2/3}$$ by $\sum_{n=0}^{4} \left(\text{binomPdf}(10, \tfrac{2}{3}, n) \times \frac{n}{10}\right)$

$= 0.042422\ldots$

$\approx \underline{\underline{0.0424}}$ (4dp)

Ex 5.3 no. 6

a) Method A

X = no. defectives

$X \sim B(10, p)$

$P(\text{batch accepted}) = P(X \leq 2)$

$= P(X=0) + P(X=1) + P(X=2)$

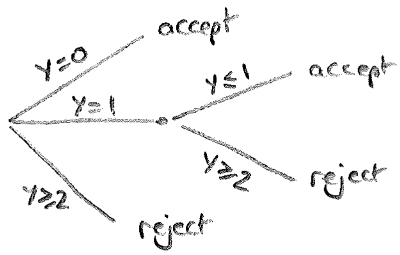
$= q^{10} + 10pq^9 + 45p^2q^8$ where $q = 1-p$.

$= (1-p)^{10} + 10p(1-p)^9 + 45p^2(1-p)^8$

Method B

Y = no. defectives

$Y \sim B(5, p)$



$P(\text{batch accepted}) = P(Y=0) + P(Y=1)P(Y \leq 1)$

$= P(Y=0) + P(Y=1)[P(Y=0) + P(Y=1)]$

$= P(Y=0) + P(Y=0)P(Y=1) + [P(Y=1)]^2$

$= q^5 + q^5 \cdot 5pq^4 + [5pq^4]^2$

$= (1-p)^5 + 5p(1-p)^9 + 25p^2(1-p)^8$

b)

		A	B
p	0.2	0.6778	0.6297
	0.5	0.0546	0.0605

from $(1-p)^{10} + 10p(1-p)^9 + 45p^2(1-p)^8 \,|\, p = \{0.2, 0.5\}$

c) when $p = 0.2$, method A has a greater chance of accepting the batch

when $p = 0.5$, method A has a lesser chance of accepting the batch. when there are more defectives.

I would opt for Method A in these instances as it has slight gains over method B, plus it will be faster/easier to administer as it takes just a single sample.