p128 Ex6C no. 1

1. $X \sim B(500, 0.002)$ $\qquad$ $Y \sim Po(500 \times 0.002)$

$Y \sim Po(1)$

a) $P(X=0) = 0.367511 \approx 0.3675$ (4dp)

$P(Y=0) = 0.367879 \approx 0.3679$ (4dp)

b) $P(X=1) = 0.368248 \approx 0.3682$ (4dp)

$P(Y=1) = 0.367879 \approx 0.3679$ (4dp)

c) $P(X=4) = 0.015252 \approx 0.0153$ (4dp)

$P(Y=4) = 0.015328 \approx 0.0153$ (4dp)

using {binompdf(500, 0.002, x), poisspdf(1, x)} | x=0

" | x=1

" | x=4

Ex 6C no. 2

$X \sim B(200, 0\cdot06)$ let $Y \sim Po(200 \times 0\cdot06)$

$Y \sim Po(12)$

So a) $P(X < 20) \approx P(Y < 20)$

$\approx P(Y \leq 19)$

$\approx 0\cdot97872$ from poissCdf (12, 0, 19)

$\approx \underline{0\cdot9787}$ (4dp)

note: exact value of $P(X<20) = 0\cdot982065$

from binomcdf (200, 0·06, 0, 19)

b) $P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - P(Y \leq 4)$

$= 1 - 0\cdot0076$

$= \underline{0\cdot9924}$ (4dp)

note: exact value of $P(X \geq 5) = 0\cdot984275$

from binomcdf (200, 0·06, 5, 20)

Ex6C no. 3

3. X = no. faulty fuses in a box of 1000

$X \sim B(1000, 0.002)$

let $Y \sim Po(2)$

a) so $P(X=2) = 0.270942$

and $P(Y=2) = 0.270671$

$\Big\}$ hence both equate to $\underline{0.271}$ (3sf)

b) $P(X \geq 1) = 0.864935$

$P(Y \geq 1) = 1 - P(Y=0)$

$= 0.864655$

$\Big\}$ hence both equate to $\underline{0.865}$ (3sf)

Ex6C no.4

P(link break under 50kg load) = 0.03

if all links are independent, then P(chain of 100 links breaks) = 1 − P(chain does not break)

= 1 − P(none of 100 links break)

$= 1 - (0.97)^{100}$

$= 0.952447$

≈ 0.9524 (4dp)

Ex6C no. 5

X = no. runs in a cricket match, in one innings

$X \sim Po(4.5)$

a) $P(X=4) = \dfrac{e^{-4.5}\, 4.5^4}{4!}$

$= 0.189808\ldots$ from poissPdf (4.5, 4)

$\approx \underline{\underline{0.1898}}$ (4dp)

b) $P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.173578\ldots$ from poissCdf (4.5, 0, 2)

$= 0.826422\ldots$

$\approx \underline{\underline{0.8264}}$ (4dp)

c) let Y = no. runs in two innings

$Y \sim Po(9)$

$P(Y \geq 6) = 1 - P(Y \leq 5)$

$= 0.884309$ from poissCdf (9, 0, 5)

$\approx \underline{\underline{0.8843}}$ (4dp)

Ex 6 C no. 6.

Binomial can be approximated by a Poission for large n, and small p (ie. $n \geq 50$, $p \leq 0.1$)

This is useful in olden times as Poisson calculations were less tedious.

It has limited benefits now as graphic calculators can do both calculations with similar ease

However, this approximation can be helpful when solving algebraically for either parameter n or p, or both, when you are given probability values.

X = no. sufferers allergic to drug

$X \sim B(8000, 0.0005)$ let $Y \sim Po(4)$

$P(X>4) \approx P(Y>4)$

$\approx 1 - P(Y \leq 4)$

≈ 0.371163 from poissCdf(4,4)

≈ 0.3712 (4dp) (exact value of $P(X \geq 4) = 0.371163$, for interest)

C = no. sufferers who develop complications

$C \sim B(8000, 0.0005 \times 0.3)$

$C \sim B(8000, 0.00015)$ $D \sim Po(8000 \times 0.00015)$

$D \sim Po(1.2)$

$P(C=2) \approx P(D=2)$

$\approx \dfrac{e^{-1.2} 1.2^2}{2!}$

≈ 0.21686 from poissPdf(1.2, 2)

$\approx \underline{\underline{0.2169}}$ (4dp)

b) E = no. who develop complications

$E \sim B(4, 0.3)$

$P(E=2) = {}^4C_2 (0.3)^2 (0.7)^2$

$= \underline{\underline{0.2646}}$. from binomPdf(4, 0.3, 2)

Answers to (a) and (b) differ as in answer (b) you are already given that 4 have an allergic reaction, whereas in (a) you still have a degree of uncertainty over how many will develop the allergic reaction.