p125 Ex6B. no. 1

$X =$ no. calls per hour, on Monday

$X \sim Po(4)$

a) $P(X \geq 6) = 1 - P(X \leq 5)$

$= 1 - 0.78513$ from poisscdf (4, 0, 5)

$\approx 0.21487\ldots$

$= \underline{\underline{0.2149}}$ (4dp)

b) $Y =$ no. calls per 2 hours

$Y \sim Po(8)$

$P(Y \leq 3) \doteqdot 0.04238$ from poisscdf (8, 0, 3)

$\approx 0.0424$ (4dp)

let $Z \sim Po(\lambda)$ $Z =$ no. of calls on Friday

we have $P(Z=0) = 0.202$

$e^{-\lambda} = 0.202$

$\lambda = -\ln(0.202)$

$\lambda = 1.59949$

$\lambda \approx 1.6.$ (1dp)

$\therefore$ Average rate of calls is $\underline{\underline{1.6 \text{ calls per hour.}}}$

Ex 6B no. 2

conditions
- events happen independently
- events happen at a constant rate per interval of time/space.

$X$ = no. windscreens needed per week

$X \sim Po(5)$

$P(\text{no more than } 7 \text{ windscreens}) = P(X \leq 7)$

$= 0.866628\ldots$ from poisscdf (5, 0, 7)

$\approx \underline{\underline{0.8666}}$ (4dp)

$Y$ = no. flaws per $0.95 m^2$

$Y \sim Po\left(\frac{48}{100} \times 0.95\right)$

$Y \sim Po(0.456)$

$P(Y<2) = P(Y \leq 1)$

$= P(Y=0) + P(Y=1)$

$= e^{-0.456} + 0.456\, e^{-0.456}$

$= 1.456\, e^{-0.456}$

$= 0.922833\ldots$

$\approx \underline{\underline{0.9228}}$ (4dp)

let $R$ = no. windscreens with fewer that 2 flaws, in sample of 5

$R \sim B(5, 0.9228)$

$P(R=3) = {}^5C_3\, (0.9228)^3 (1-0.9228)^2$

$= 0.046799\ldots$

$\approx \underline{\underline{0.0468}}$ (4dp)

Ex6B no.3

$X$ = no. spare parts used per week

$X \sim Po(5)$

a) $P(X=5) = \frac{e^{-5}5^5}{5!}$

$= 0{\cdot}175467...$

$\approx \underline{0{\cdot}1755}$ (4dp)

b) $P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - 0{\cdot}440493$ from poisscdf (5,0,4)

$= 0{\cdot}559507...$

$\approx \underline{0{\cdot}5595}$ (4dp)

c) let $Y$ = no. spare parts used in 3 weeks

$Y \sim Po(15)$

$P(Y=15) = \frac{e^{-15}15^{15}}{15!}$

$= 0{\cdot}102436..$

$\approx \underline{0{\cdot}1024}$ (4dp)

d) $P(Y \geq 15) = 1 - P(Y \leq 14)$

$= 1 - 0{\cdot}465654..$

$\approx 0{\cdot}534346...$

$\approx \underline{0{\cdot}5343}$ (4dp)

e) P(exactly 5 used in 3 successive weeks) $= P(X=5)P(X=5)P(X=5)$

$= (0{\cdot}1755)^3$

$= 0{\cdot}005402...$

$\approx \underline{0{\cdot}0054}$ (4dp)

if $X \sim Po(5)$, we need to determine stock level $x$, so that $P(X > x)$ is so small that it will only happen - on average - once out of 52 occurrences

ie. $P(X > x) = \frac{1}{52}$

$P(X \leq x) = \frac{51}{52}$

so $\underline{x = 10}$ from nSolve $\left(\text{poisscdf}(5,0,x) = \frac{51}{52}, x\right) \mid x > 0$

check: $P(X \leq 10) = 0{\cdot}986305$

$\frac{51}{52} = 0{\cdot}980769$

$P(X \leq 9) = 0{\cdot}968172$ ✓

Ex6B no. 4

$X$ = mean daily demand for vaccuum cleaners

$X \sim Po\ (2.6)$

a) $E(\text{income}) = E(X) \times 5$

$= 2.6 \times 5$

$= £13.$

b) i) $P(X=0) = e^{-2.6}$

$= 0.074274$

$\approx 0.0743$ (4dp)

ii) $P(X=1) = \dfrac{2.6^1 e^{-2.6}}{1!}$

$= 0.193111...$

$\approx 0.1931$ (4dp)

iii) $P(X=2) = \dfrac{2.6^2 e^{-2.6}}{2!}$

$= 0.251045...$

$\approx 0.2510$ (4dp)

iv) $P(X \geq 3) = 1 - P(X \leq 2)$

$= 1 - 0.51843..$

$= 0.48157...$

$\approx 0.4816$ (4dp)

c)

| income, $y$ | 0 | 5 | 10 | 15 |
|---|---|---|---|---|
| $P(Y=y)$ | 0.0743 | 0.1931 | 0.2510 | 0.4816 |

s $E(Y) = \sum y P(Y=y)$

$= 10.6996...$

so mean daily income is £10.70.

With an unlimited number of cleners, expect to have daily income of £13

However, with cost of £2 to service this, net income expected is £11

With a stock of 3 machines, expected income is £10.70

so, using the service of the nearby large store, expected increase in revenue is 30p per day

This amounts to, over a 365 day year, to be an extra £109.50

So, yes, it is probably worth it, especially if the large store delivers them!