CIMT Statistics p185 Ex 9.6

1. $n = 250$

$\bar{x} = 57.1$

$s = 11.8$

a) we use the large sample size, $n=250$, to allow us to approximate population σ with s, without using t-distn

if $E(X) = \mu$ and $Var(X) = \sigma^2$,

then $\bar{X} \approx N(\mu, \frac{\sigma^2}{n})$ by CLT, as n is large

so st. error of mean = st. dev of $\bar{X}$

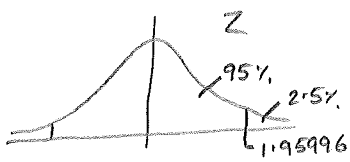
$$= \sqrt{\frac{\sigma^2}{n}}$$

$$\approx \sqrt{\frac{s^2}{250}}$$

$$= \frac{11.8}{\sqrt{250}}$$

$$= 0.746298..$$

$$\approx 0.746 \quad (3dp)$$



b) 95% CI for μ is $\bar{x} \pm 1.95996\sqrt{\frac{s^2}{250}}$ where $1.95996 = \text{invNorm}(0.975$

$$= 57.1 \pm 1.95996 \times 0.746298$$

$$= (55.6373, 58.5627)$$ ← from $57.1 + \{-1, 1\} \times 1.96 \times 0.746$ on Nspire.

$$\approx (55.64, 58.56) \quad \text{2dp}$$

2. 99% confident to be $> x$ mL.

let X = volum of lemonade in can

$\text{Var}(X) = 3.2^2$

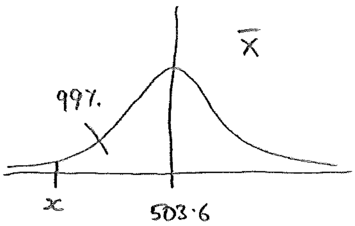
$\bar{x} = 503.6$ when $n = 50$

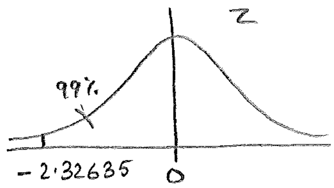
as n is large and we are interested in mean, we use CLT on Z-distn (as n is large, no need to use t distn)

so $\bar{X} \approx N\left(\mu, \frac{\sigma^2}{n}\right)$

we estimate μ with $\bar{x}$, and σ^2 with 3.2^2

$\Rightarrow \bar{X} \sim N\left(503.6, \frac{3.2^2}{50}\right)$





from invNorm(0.01)

so $x = 503.6 - 2.32635 \times \sqrt{\frac{3.2^2}{50}}$

$x = 502.547..$

so $x \approx 502.5$ mL.

from invT(0.01, 49) $= t_{49, 0.01}$

IF, we use t-distn, then $x = 503.6 - 2.40489 \times \sqrt{\frac{3.2^2}{50}}$

≈ 502.512

≈ 502.5 mL, so no difference in final value, to 1dp.

3.

X = mass of butter.

X is normally distributed

X_u = mass unsalted butter $\qquad\qquad$ X_S = mean salted butter

$X_u \sim N(\mu_u, 8{\cdot}45^2)$ $\qquad\qquad$ $X_S \sim N(225{\cdot}38, 8{\cdot}45^2)$

we have sample, $n = 12$

sample $\bar{x} = 222{\cdot}667$

so $\bar{X}_u \sim N\left(\mu_u, \frac{8{\cdot}45^2}{12}\right)$

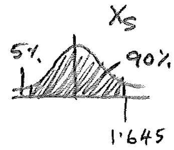
where $\bar{X}_u$ = mean mass of unsalted butter

so 95% CI for $\mu_u = \bar{x} \pm 1{\cdot}96\sqrt{\frac{8{\cdot}45^2}{12}}$

$= 222{\cdot}667 \pm 1{\cdot}96 \times \sqrt{5{\cdot}95021}$

$= (217{\cdot}886, 227{\cdot}448)$

$\approx \underline{(217{\cdot}9, 227{\cdot}4)}$ to 1dp.



from invNorm (0.95)

For salted butter, 90% will lie in the interval $225{\cdot}38 \pm 1{\cdot}645 \times 8{\cdot}45$

$= (211{\cdot}481, 239{\cdot}279)$

$\approx \underline{(211{\cdot}5, 239{\cdot}3)}$ (to 1dp)

we seek n such that width of CI is 6

$\Rightarrow$ half of width ≤ 3

so $\quad 1{\cdot}96\sqrt{\frac{8{\cdot}45^2}{n}} \leq 3$

$\sqrt{\frac{8{\cdot}45^2}{n}} \leq 1{\cdot}53064$

$\frac{8{\cdot}45^2}{n} \leq 2{\cdot}34286$

$n \geq 30{\cdot}4766$

as n is an integer, you would need a sample of size $\underline{31}$

We would use the same sample size when sampling unsalted packs of butter, as the above calculation requires only the population's standard deviation, which has been given as being the same for both types of butter.

4 we use midpoints of intervals to estimate $\bar{x}$ and s.

x	5.61	5.63	5.65	5.67	5.69	5.71	5.73	5.75	5.77	5.79
f	1	3	5	5	8	20	24	16	12	6

on TI-Nspire. ← seq(5.59+n0.02, n, 1, 10)

so $\bar{x} = 5.7232$

$s_{n-1} = 0.039921$

so if X = length of rods

$E(X) = \mu$

$Var(X) = \sigma^2$

and if $n=100$, by CLT $\bar{X} \approx N(\mu, \frac{\sigma^2}{n})$

we estimate σ with s_{n-1} so standard error of mean = st. dev of $\bar{X}$

$$\doteqdot \sqrt{\frac{s^2}{n}}$$

$$= 0.003992.$$

so 95% CI for μ $= 5.7232 \pm 1.96 \times \sqrt{\frac{0.039921^2}{100}}$

$\approx (5.71538, 5.73102)$

$\approx (5.715, 5.731)$ (to 3dp)

we seek n so that $1.96\sqrt{\frac{0.039921^2}{n}} < 0.002$

$$\sqrt{\frac{0.039921^2}{n}} < 0.001020$$

$$\frac{0.039921^2}{n} < 0.000001041$$

$$1530.58 < n$$

so sample size needs to be at least <u>1531</u>

<u>note</u>: solutions at back of eBook use $s_n = 0.039721$ as estimate for σ, presumable due to large sample size, $n=100$.
Technically, the above solution is more correct as s_{n-1} is used to estimate the parent population parameter from the sample.

5. X = weight of impurity per 100g, in mg.

$X \sim N(\mu, 3.2^2)$

a) $n = 12$

$\bar{x} = 7.75$

$S_{n-1} = 2.89969$

$X \sim N(\mu, 3.2^2)$

$\bar{X} \sim N\left(\mu, \frac{3.2^2}{12}\right)$

i) so 95% CI for $\mu = \bar{x} \pm 1.96 \times \sqrt{\frac{3.2^2}{12}}$

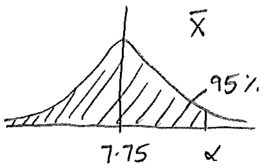
$= 7.75 \pm 1.96 \times 0.92376$

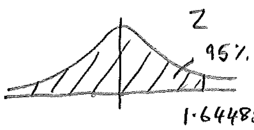
$= (5.93946, 9.56054)$

$\approx \underline{(5.94, 9.56)}$ to 2dp

On TI-Nspire:
MENU
Statistics
Confidence Intervals
z Interval

ii)





from invNorm(0.95)

so $\alpha = 7.75 + 1.64485\sqrt{\frac{3.2^2}{12}}$

$\alpha = 9.26945$

$\alpha \approx \underline{\underline{9.27}}$

Given the context of the question, having too little impurity would not be a problem, and hence no need for a lower bound to the confidence interval.

iii) if $X \sim N(7.75, 3.2^2)$, then 90% of weights will lie in the interval

$7.75 \pm 1.64485\sqrt{3.2^2}$

$= (2.48647, 13.0135)$

$\approx \underline{\underline{(2.49, 13.01)}}$ to 2dp)

b) we seek n such that $1.96\sqrt{\frac{3.2^2}{n}} < 1.5$

$\frac{3.2^2}{n} < 0.585715$

$n > 17.4829$

so the scientist should take at least $\underline{\underline{\text{18 samples}}}$.

6. X = survival time in days.

a) $n = 10$

$\bar{x} = 391.3$

$s_{n-1} = 251.723$ from Nspire.

b) $X \sim N(\mu, 240^2)$

So $\bar{X} \sim N(\mu, \frac{240^2}{10})$ where $\bar{X}$ = mean survival time in days.

So 90% CI for $\mu = 391.3 \pm 1.645\sqrt{\frac{240^2}{10}}$

$= 391.3 \pm 124.836$

$= (266.464, 516.136)$

$\approx (266.5, 516.1)$ (to 1dp)

7. X_w = mass of white bars of soap

$X_w \sim N(176.2, 6.46^2)$

X_p = mass of pink bars of soap.

$X_p \sim N(\mu, 6.46^2)$

$n = 12$

$\bar{x} = 174.5$

$s_{n-1} = 6.45967$

so $X \sim N(\mu, 6.46^2)$

$\bar{X} \sim N\left(\mu, \frac{6.46^2}{12}\right)$

so 95% CI for $\mu = 174.5 \pm 1.96\sqrt{\frac{6.46^2}{12}}$

$= 174.5 \pm 3.65502$

$= (170.845, 178.155)$

$\approx \underline{\underline{(170.85, 178.16)}}$ (2dp)

90% of white soap will weigh $176.2 \pm 1.645 \times 6.46$

$= (165.574, 186.826)$

$\approx \underline{\underline{(165.6, 186.8)}}$ to 1dp.

so $X_p \sim N(\mu, 6.46^2)$

and 95% CI for μ is $(170.85, 178.16)$

let Y_p = net profit from selling a pink bar of soap

so $Y_p = 32 - (15 + 0.065X_p)$

$Y_p = 17 - 0.065X_p$

$\therefore$ 95% CI for $Y_p = 17 - 0.065 \times (170.85, 178.16)$

$= (5.89508, 5.41992)$

so for 1 bar, expected profit 95% of time is between 5.41992p and 5.89508p

$\therefore$ for 9000 bars, expected profit 95% CI is $(5.41992 \times 9000, 5.89508 \times 9000)$ pence

$= (48779.3, 53055.7)$ pence

$= \underline{\underline{(£487.79, £530.56)}}$ to 2dp)

8 X_G = mass granulated sugar $\qquad$ X_C = mass caster sugar

$X_G \sim N(1022.51, 8.21^2)$ $\qquad$ $X_C \sim N(\mu, 8.21^2)$

90% of X_G will lie in $1022.51 \pm 1.645 \times 8.21$

$= (1009.01, 1036.01)$

$\approx \underline{(1009.0, 1036.0)}$ (to 1dp)

for X_C sample of $n=10$ gave $\bar{x}_c = 1032.4$

$s_{n-1} = 21.7879$

so $X_C \sim N(\mu, 8.21^2)$

$\bar{X}_C \sim N\left(\mu, \frac{8.21^2}{10}\right)$ where $\bar{X}_C$ = mean mass of 10 caster sugar bags.

so 99% CI for $\mu = 1032.4 \pm z_{0.995} \times \sqrt{\frac{8.21^2}{10}}$

$= 1032.4 \pm 2.57583 \times \sqrt{\frac{8.21^2}{10}}$ $\qquad$ from invNorm (0.995)

$= 1032.4 \pm 6.68745$

$= (1025.71, 1039.09)$

$\approx \underline{(1025.7, 1039.1)}$ to 1dp)

let Y_c = net profit from 1 bag of caster sugar

$Y_c = 65 - (32 + 0.023 X_c)$

$Y_c = 33 - 0.023 X_c$

so 99% CI for Y_c is $33 - 0.023 \times (1025.7, 1039.1)$

$= (9.10099, 9.40861)$

so 99% CI for $10000 Y_c$ is $10000 \times (9.10099, 9.40861)$ pence

$= \underline{(£910.10, £940.86)}$