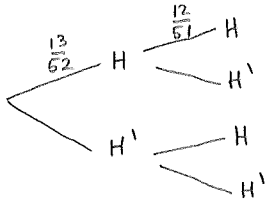
CIMT Stats p19 Ex1E

1, a) P(heart then heart)

$= \frac{13}{52} \times \frac{12}{51}$

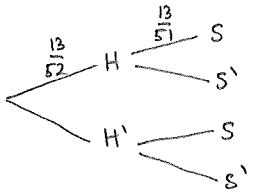
$= \frac{1}{17}$



b) P(heart then spade)

$= \frac{13}{52} \times \frac{13}{51}$

$= \frac{13}{204}$

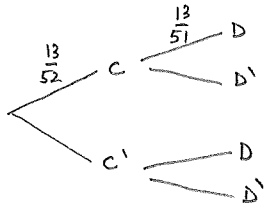


c) P(second is diamond | first is a club)

$= \frac{P(\text{second is diamond and first is a club})}{P(\text{first is a club})}$

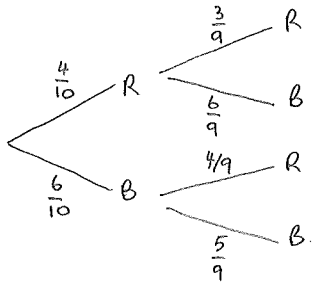
$= \frac{\frac{13}{51} \times \frac{13}{52}}{\frac{13}{52}}$

$= \frac{13}{51}$



Ex1E cont.

2. 4 red
6 black
without replacement



a) P(second is red | first is red)

$= \frac{3}{9}$ ← no. reds left
← no. counters left.

$= \frac{1}{3}$.

b) P(both red)

$= \frac{4}{10} \times \frac{3}{9}$

$= \frac{2}{5} \times \frac{1}{3}$.

$= \frac{2}{15}$.

c) P(counters different)

= P(red then blue) + P(blue then red)

$= \frac{4}{10} \times \frac{6}{9} + \frac{6}{10} \times \frac{4}{9}$

$= \frac{48}{90}$

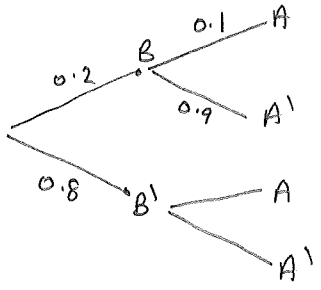
$= \frac{8}{15}$.

Ex 1E cont.

3. $P(A) = 0.6$

$P(B) = 0.2$

$P(A|B) = 0.1$



∴ a) $P(A \cap B) = P(B) \times P(A|B)$

$= 0.2 \times 0.1$

$= \underline{0.02}$.

b) $P(\text{at least one occurs}) = 1 - P(\text{neither occurs})$

we need to work out $P(A'|B')$

now $P(A) = P(B \cap A) + P(B' \cap A)$

$0.6 = 0.2 \times 0.1 + 0.8 \times P(A|B')$

$0.6 = 0.02 + 0.8 \times P(A|B')$

$0.58 = 0.8\ P(A|B')$

$P(A|B') = \frac{0.58}{0.8}$

$= 0.725$.

∴ $P(A'|B') = 1 - 0.725$

$= 0.275$

∴ $P(\text{at least one occurs}) = 1 - P(B' \cap A')$

$= 1 - 0.8 \times 0.275$

$= \underline{0.78}$.

c) $P(\text{exactly one event occurs}) = P(B \cap A') + P(B' \cap A)$

$= 0.2 \times 0.9 + 0.8 \times 0.725$

$= \underline{0.76}$.

d) $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$= \frac{0.2 \times 0.1}{0.2 \times 0.1 + 0.8 \times 0.725}$

$= \frac{0.02}{0.6}$

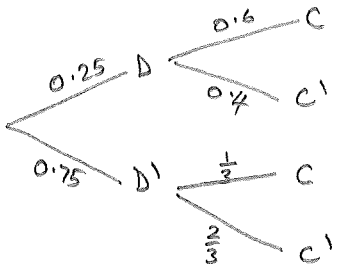
$= \underline{\frac{1}{30}}$.

Ex 1E cont.

4. $P(\text{cat}) = 0.4$

$P(\text{dog}) = 0.25$

$P(\text{both}) = 0.15$



$P(D \cap C) = P(D) \times P(C|D)$

$0.15 = 0.25 \times P(C|D)$

$P(C|D) = \frac{0.15}{0.25}$

$P(C|D) = 0.6$

$P(C) = P(D \cap C) + P(D' \cap C)$

$0.4 = 0.25 \times 0.6 + 0.75\, P(C|D')$

$0.4 = 0.15 + 0.75\, P(C|D')$

$0.25 = 0.75\, P(C|D')$

$P(C|D') = \frac{0.25}{0.75}$

$P(C|D') = \frac{1}{3}$.

a) $P(\text{owns a dog or a cat})$

$= 1 - P(\text{owns neither})$

$= 1 - P(D' \cap C')$

$= 1 - 0.75 \times \frac{2}{3}$

$= 1 - \frac{3}{4} \times \frac{2}{3}$.

$= \frac{1}{2}$.

b) $P(\text{owns a dog or a cat, but not both})$

$= P(D \cap C') + P(D' \cap C)$

$= 0.25 \times 0.4 + 0.75 \times \frac{1}{3}$.

$= 0.35$.

c) $P(D|C)$

$= \frac{P(D \cap C)}{P(C)}$

$= \frac{0.25 \times 0.6}{0.4}$

$= 0.375$.

d) $P(C'|D)$

$= \frac{P(C' \cap D)}{P(D)}$

$= \frac{0.25 \times 0.4}{0.25}$.

$= 0.4$.