p120 Ex6A no. 1.

1 a) $X \sim Po(3)$

$$P(X=2) = \frac{e^{-3}3^2}{2!}$$

$$= \frac{9}{2e^3}$$

$$\doteq 0.224042\ldots$$

$$= \underline{\underline{0.2240}} \text{ (4dp)}$$

b) $$P(X=3) = \frac{e^{-3}3^3}{3!}$$

$$= \frac{27}{6e^3}$$

$$= \frac{9}{2e^3}$$

$$= \underline{\underline{0.2240}} \text{ (4dp)}$$

c) $P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - 0.815263$ from poissCdf(3,0,4)

$= 0.184737$

$= \underline{\underline{0.1847}}$ (4dp)

d) $P(X < 3) = P(X \leq 2)$

$= 0.42319\ldots$ from poisscdf(3,0,2)

$= \underline{\underline{0.4232}}$ (4dp)

Ex6A no. 2

$X \sim Po(\lambda)$

$P(X=4) = \dfrac{\lambda^4 e^{-\lambda}}{4!}$

$P(X=3) = \dfrac{\lambda^3 e^{-\lambda}}{3!}$

$P(X=4) = 3 \cdot P(X=3)$

$\dfrac{\lambda^4 e^{-\lambda}}{4!} = 3 \cdot \dfrac{\lambda^3 e^{-\lambda}}{3!}$

$\div \lambda^3 e^{-\lambda}$

$\dfrac{\lambda}{24} = 3 \cdot \dfrac{1}{6}$

$\lambda = 24 \times \frac{1}{2}$

$\underline{\lambda = 12}$

so $P(X=5) = \dfrac{e^{-12}\, 12^5}{5!}$

$= 0.012741\ldots$

$\approx \underline{\underline{0.0127}}$ (4dp)

Ex6A no. 3.

$X \sim Po(\lambda)$

$$P(X=0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda}$$

so $e^{-\lambda} = 0.323$

$$\ln(e^{-\lambda}) = \ln(0.323)$$

$$-\lambda = \ln(0.323)$$

$$\lambda = -\ln(0.323)$$

$$\lambda = 1.1301....$$

$$\underline{\lambda \approx 1.13} \text{ (2dp)}$$

$$\text{so } P(X=3) = \frac{e^{-1.13}\, 1.13^3}{3!}$$

$$= 0.077684..$$

$$\approx \underline{0.0777} \text{ (4dp)}.$$

Ex6A no. 4.

if $X \sim Po(\lambda)$ and $P(X=0) = 0.368$

then $e^{-\lambda} = 0.368$

$\ln(e^{-\lambda}) = \ln(0.368)$

$\lambda = -\ln(0.368)$

$\lambda = 0.999672...$

$\lambda \approx 1$

so, if $\lambda = 1$, will this generate the same probabilities

well, if $\lambda = 1$, then

x	0	1	2	3	4	5	6
$P(X=x)$	0.368	0.368	0.184	0.061	0.015	0.003	0.001

using round(poisspdf (1, {0,1,2,3,4,5,6}), 3)

and thus $P(X \geq 7) = 0.000083$

so, yes, the figures could come from a Poi(1) distribution.

ExGA no.5.

$X \sim Po(2)$

$Y \sim Po(3)$

$Z \sim Po(5)$

a) $P(X+Y=0) = P(X=0)P(Y=0)$ assuming X & Y are independent

$= e^{-2} \cdot e^{-3}$

$= e^{-5}$

$= P(Z=0)$

$= 0.006738\ldots$

$= \underline{0.0067}$ (4dp)

b) $P(X+Y=1) = P(X=0)P(Y=1) + P(X=1)P(Y=0)$ assuming X & Y are independent

$= e^{-2} \cdot \dfrac{3^1 e^{-3}}{1!} + \dfrac{2^1 e^{-2}}{1!} \cdot e^{-3}$

$= 3e^{-5} + 2e^{-5}$

$= 5e^{-5}$

$= \dfrac{5^1 e^{-5}}{1!}$

$= P(Z=1)$

$= 0.03369\ldots$

$\approx \underline{0.0337}$ (4dp)

c) $P(Z=0) = \underline{0.0067}$ (4dp) - from part (a)

d) $P(Z=1) = \underline{0.0337}$ (4dp) - from part (b)

e) $P(X+Y \leq 2) = P(X=0)P(Y=0) + P(X=0)P(Y=1) + P(X=0)P(Y=2) + P(X=1)P(Y=0) + P(X=1)P(Y=1) + P(X=2)P(Y=0)$

$= e^{-2}e^{-3} + e^{-2} \cdot 3e^{-3} + e^{-2} \cdot \dfrac{9e^{-3}}{2!} + 2e^{-2} \cdot e^{-3} + 2e^{-2} 3e^{-3} + \dfrac{2^2 e^{-2}}{2!} \cdot e^{-3}$

$= e^{-5}\left[1 + 3 + \frac{9}{2} + 2 + 6 + 2\right]$

$= \dfrac{37}{2e^5}$

$= 0.124652\ldots$

$= \underline{0.1247}$ (4dp)

f) $P(Z \leq 2) = \underline{0.1247}$ (4dp) from poisscdf(5,0,2).