p137 Ex 7A no. 1

1a) $f(x) = \frac{1}{2}(x^2+4) \quad 0<x<1$

Suitable if $\int_0^1 f(x)\,dx = 1$

So $\int_0^1 f(x)\,dx = \int_0^1 \frac{1}{2}(x^2+4)\,dx$

$= \frac{1}{2}\int_0^1 (x^2+4)\,dx$

$= \frac{1}{2}\left[\frac{1}{3}x^3+4x\right]_0^1$

$= \frac{1}{2}\left(\left(\frac{1}{3}+4\right)-(0+0)\right)$

$= \frac{1}{2}\cdot\frac{13}{3}$

$= \frac{13}{6}$

$\neq 1$

So, not a suitable pdf.

b) $f(x) = \frac{1}{2} \quad 2<x<4$

$\int_2^4 f(x)\,dx = \int_2^4 \frac{1}{2}\,dx$

$= \left[\frac{1}{2}x\right]_2^4$

$= \frac{1}{2}\cdot 4 - \frac{1}{2}\cdot 2$

$= 2-1$

$= 1$ ✓

Yes, suitable pdf.

c) $f(x) = \frac{x}{4} \quad 1<x<3$

$\int_1^3 f(x)\,dx = \int_1^3 \frac{1}{4}x\,dx$

$= \left[\frac{1}{8}x^2\right]_1^3$

$= \frac{1}{8}(3^2-1^2)$

$= \frac{1}{8}(9-1)$

$= 1$ ✓

Yes, suitable pdf

d) $f(x) = \frac{x}{6}+\frac{1}{12} \quad 0<x<3$

So $\int_0^3 f(x)\,dx = \int_0^3 \left(\frac{x}{6}+\frac{1}{12}\right)dx$

$= \left[\frac{x^2}{12}+\frac{1}{12}x\right]_0^3$

$= \frac{1}{12}\left[x^2+x\right]_0^3$

$= \frac{1}{12}\left((3^2+3)-(0+0)\right)$

$= \frac{12}{12}$

$= 1$

So, yes a suitable pdf.

e) $f(x) = \frac{1}{2}(2x-1) \quad 0<x<2$

So $\int_0^2 f(x)\,dx = \int_0^2 \frac{1}{2}(2x-1)\,dx$

$= \frac{1}{2}\int_0^2 (2x-1)\,dx$

$= \frac{1}{2}\left[x^2-x\right]_0^2$

$= \frac{1}{2}\left[(2^2-2)-(0^2-0)\right]$

$= \frac{1}{2}(4-2-0)$

$= \frac{2}{2}$

$= 1$

So, yes a suitable pdf.

[Verified on nspire]

Ex 7A no. 2

$f(x) = A(x^2+4) \quad 0<x<1$

valid pdf if $\int_0^1 f(x)\,dx = 1$

$$\int_0^1 A(x^2+4)\,dx = 1$$

$$A\int_0^1 (x^2+4)\,dx = 1$$

$$\left[\tfrac{1}{3}x^3 + 4x\right]_0^1 = \frac{1}{A}$$

$$\left(\tfrac{1}{3}+4\right) - (0+0) = \frac{1}{A}$$

$$\frac{13}{3} = \frac{1}{A}$$

$$A = \frac{3}{13}.$$