p162 Ex8D no. 1

1. $X$ = weight of bag of sugar

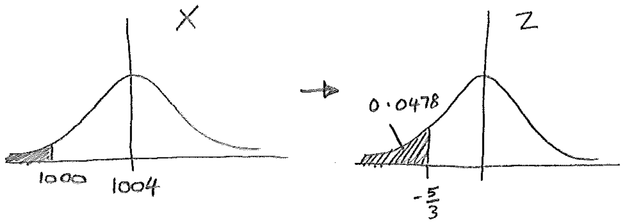
$E(X) = 1004$

$Var(X) = 2.4^2$

assume $X$ distributed normally

$X \sim N(1004, 2.4^2)$

$$
\begin{aligned}
P(\text{underweight}) &= P(X < 1000) \\
&= P\left(Z < \frac{1000 - 1004}{2.4}\right) \\
&= P\left(Z < -\frac{4}{2.4}\right) \\
&= P\left(Z < -\frac{5}{3}\right) \\
&\doteqdot 0.04779033\ldots \\
&\approx \underline{\underline{0.0478}} \quad (4dp)
\end{aligned}
$$




from normCdf$(-9E99, -\frac{5}{3})$

So proportion under weigh $\approx \underline{\underline{4.8\%}}$

Ex 8D no. 2.

$20.5 < \text{accept} < 20.6$

$X$ = dimension of part

$X \sim N(20.56, 0.02^2)$

$$
\begin{aligned}
P(\text{reject}) &= 1 - P(\text{accept}) \\
&= 1 - P(20.5 < X < 20.6) \\
&= 1 - P\left(\frac{20.5 - 20.56}{0.02} < Z < \frac{20.6 - 20.56}{0.02}\right) \\
&= 1 - P(-3 < Z < 2) \\
&= 1 - 0.9759\ldots \quad \text{from normCdf}(-3, 2) \\
&= 0.024100029\ldots \\
&\approx 0.0241 \quad (4\text{dp})
\end{aligned}
$$

let $Y$ = no. rejected from 1000 parts.

$Y \sim B(1000, 0.0241)$

$$
\begin{aligned}
E(Y) &= 1000 \times 0.0241 \\
&= 24.1
\end{aligned}
$$

We would expect to reject <u>about</u> 24 of them.

Ex 8D no. 3.

$T$ = two year old bouyancy aid weight support

$T \sim N(6, 0.8^2)$

$F$ = five year old bouyancy aid weight support

$F \sim N(4.5, 1^2)$

a) $P(\text{usable}) = P(T > 5)$

$= P\left(Z > \frac{5-6}{0.8}\right)$

$= P(Z > -1.25)$

$= 0.89435$ from normCdf $(-1.25, 9E99)$

Out of 24 aids, we expect $24 \times 0.89435$

$= 21.4644$ to be usable

Hence about 21 or 22 of them should still be useable

b) $P(\text{usable}) = P(F > 5)$

$= P\left(Z > \frac{5-4.5}{1}\right)$

$= P(Z > 0.5)$

$= 0.308538$ from normCdf $(0.5, 9E99)$

Out of 32 aids, we expect $32 \times 0.308538$

$= 9.8732$ to be usable

Hence about 9 or 10 of them should still be useable.

Ex8D no.4

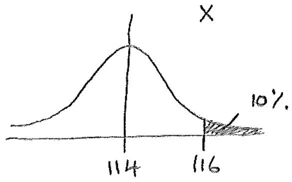
$X$ = weight of sack of potatoes

$E(X) = 114$ lb

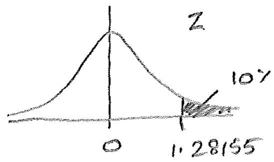
10% bags > 116 lb.

$Var(X) = \sigma^2$

we assume $X$ is normally distributed.




so $P(Z > 1.28155) = 0.1$




from invNorm(0.9)

so $\dfrac{116-114}{\sigma} = 1.28155$

$$\sigma = \frac{2}{1.28155}$$

$$\sigma = 1.56061$$

$$\underline{\underline{\sigma \approx 1.56}}$$

$E(X)$ is now 113 lb.

so, again assuming normality, $X \sim N(113, 1.56^2)$

$$P(X > 116) = P\left(Z > \frac{116-113}{1.56}\right)$$

$$= P(Z > 1.92233\ldots)$$

$$= 0.027282\ldots$$

from normCdf(1.92233, 9E99)

$$\approx 0.0273 \text{ (4dp)}$$

$$\underline{\underline{\approx 2.7\%}}$$

Ex 8D no. 5

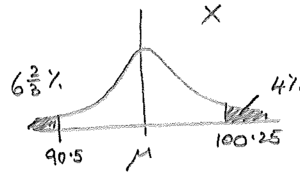
$X$ = weight of soap bar.

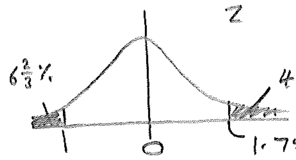
a) $P(X < 90.5) = 6\frac{2}{3}\%$

$P(X > 100.25) = 4\%$

we assume $X$ is normally distributed

$X \sim N(\mu, \sigma^2)$




$-1.50109$ from invNorm$\left(\frac{20}{3} \times \frac{1}{100}\right)$

$1.75069$ from invNorm$(0.96)$

so

$$\frac{90.5 - \mu}{\sigma} = -1.50109$$

$$\frac{100.25 - \mu}{\sigma} = 1.75069$$

using Nspire LinSolve $\left(\begin{cases} \frac{90.5 - m}{1} = -1.50109s \\ \frac{100.25 - m}{1} = 1.75069s \end{cases}, \{m, s\}\right)$ we get $\mu = 95.0008$, $\sigma = 2.99837$

so mean $\approx$ $\underline{\underline{95g}}$ and st. dev $\approx$ $\underline{\underline{3g}}$.

b) using these values, and still assuming $X$ to be normally distributed

$X \sim N(95, 3^2)$

$P(X < 88) = P\left(Z < \frac{88 - 95}{3}\right)$

$= P\left(Z < -\frac{7}{3}\right)$ from normCdf$\left(-9E99, -\frac{7}{3}\right)$

$= 0.009815\ldots$

$\approx 0.0098$ (4dp)

$\approx \underline{\underline{0.98\%}}$.

Ex 8D no. 6.

$X$ = lifetime of bulbs.

$P(X > 500) = 0.05$

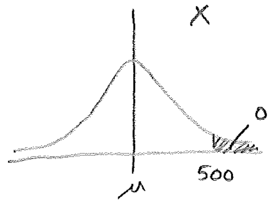
then improvement mean $E(X) \to E(X)+50$.

new test $P(X>500) = 0.2$

assume $X$ is normally distributed

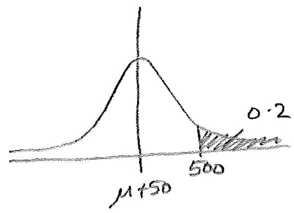
BEFORE

$X \sim N(\mu, \sigma)$




AFTER

$X \sim N(\mu+50, \sigma)$




now $\dfrac{500-\mu}{\sigma} = 1.64485$ (from invNorm (0.95))

and $\dfrac{500-(\mu+50)}{\sigma} = 0.841621$ (from invNorm (0.8))

use TI-Nspire Linsolve $\left(\begin{cases} 500-m = 1.64485s \\ 500-(m+50) = 0.841621s \end{cases}, \{m, s\}\right)$ to give $\mu = 397.61$

$\sigma = 62.2485$

so original process had mean of $\underline{397.6}$ hrs (1dp)

and st. dev of $\underline{\underline{62.2}}$ hrs (1dp).